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A Partial Local Search Algorithm for the Design of Multiplierless FIR Digital Filters with CSD Coefficients and Its FPGA Implementation

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Abstract: This paper proposes a method for designing the multiplierless finite impulse response (FIR) digital filters over the Canonic Signed-Digit (CSD) coefficient space. The proposed method is a novel local search algorithm with respect to the frequency of appearance of signed-power-of-two (SPT) terms. The proposed algorithm is about 2 times as fast as the conventional local search algorithms. Although this algorithm is intended to reduce the number of partial SPT terms, it can be used to design a FIR filter which has small number of SPT terms as the conventional algorithms. The filter operation and its performance are evaluated by using a field programmable gate array (FPGA). The implementation example shows that the area-time product of the proposed 28th-order lowpass filter is about 2–17% smaller than those of the other local search algorithms. In addition, the other implementation example shows that the area-time product of the proposed 31st-order Hilbert transformer is about 85% smaller than that of a 2's complement implementation.

1. Introduction

In VLSI implementations, a general multiplier element is very costly. From this reason, it is attractive to carry out the multiplication by using shifts and adds. The shifts can be realized by using hard-wired shifters and hence they are essentially free.

During the past two decades, numerous algorithms for designing the multiplierless FIR filters have been proposed [1]–[10]. They include the mixed-integer linear programmings (MILPs) [1]–[4], the genetic algorithms (GAs) [5], [6] and the local search algorithms [7]–[10]. The MILPs and GAs require a long computation time, which limits its usefulness to designing filters whose coefficient length is relatively short. On the other hand, the local search algorithms have the shortest computation time of three algorithms described in the above, however, these methods are inferior to MILPs and GAs in the light of its accuracy. In view of computer design and hardware implementation, the local search algorithms have been found to perform nearly as well as the MILPs while requiring substantially less computational time. Consequently, even if all the coefficients cannot meet the optimal solution, the local search algorithm is a more effective means of the multiplierless FIR filter design.

In this paper, we describe a design method for the optimized FIR digital filters over the CSD coefficient space. The proposed algorithm is performed through two steps. First, a prototype optimal FIR filter is designed by using the Remez-exchange algorithm. The second step involves finding the filter parameter such that the resulting filter meets the given criteria with the simplest coefficient representation forms. This approach is based on the local search algorithm. In addition, design examples and FPGA implementations show the usefulness of the proposed algorithm.

2. Background

2.1 Linear-Phase FIR Filters

The zero-phase frequency response of a linear-phase Nth-order FIR filter can be expressed as

$$H(\omega) = \sum_{n=0}^{M} h(n)\ \mathrm{Trig}(\omega, n), \tag{1}$$

where $h(n)$ are filter coefficients and $\mathrm{Trig}(\omega,n)$ is an appropriate trigonometric function depending on whether N is odd or even and whether the impulse response is symmetrical or antisymmetrical. Here, $M = N/2$ if N is even, $M = (N-1)/2$ if N is odd.

The general form for expressing coefficient values as a sum of SPT terms is given by

$$h(n) = \sum_{m=1}^{L} a_{m,n}\ 2^{-m}, \tag{2}$$

where $a_{m,n} \in \{\bar{1}, 0, 1\}$. Here, $\bar{1}$ denotes -1. In this representation form, maximum allowable wordlength is L-bits. A minimum representation has the minimum required number of SPT terms. One minimum representation is the CSD code representation. Here, two SPT terms can not be adjacent. The total number of SPT terms (#SPT) in Eq.(2) is

$$\#\mathrm{SPT} = \sum_{n=0}^{M} \sum_{m=1}^{L} \left(a^{+}_{m,n} + a^{-}_{m,n}\right). \tag{3}$$

where $a^+_{m,n}$ and $a^-_{m,n}$ are positive and negative nonzero bits, respectively. In some applications, it may be attractive to limit the number of SPT terms for each coefficients. This may be the case for a transposed form FIR filter which has the critical path without pipeline determined by the coefficient with most SPT terms. Of course, the filter can be pipelined, but the number of pipeline stages required for a given sample rate is also dependent on the number of SPT terms for a coefficient. In the optimization, this can be handled by adding the following constraint

$$\sum_{m=1}^{L}\left(a^+_{m,n}+a^-_{m,n}\right)\leq S_{max}, \qquad (4)$$

where S_{max} is the maximum number of SPT terms per coefficient.

2.2 The Average Appearance Ratio of SPT Terms

The average appearance ratio of SPT terms allocated to the m-th digit of all M-digit CSD numbers [4] is given by

$$g(m)=\frac{6\{1-(-0.5)^m\}\{1-(-0.5)^{M-m+1}\}}{(6M+4)-(3M+4)(-0.5)^M}. \qquad (5)$$

Because the designed FIR filters have many small coefficients with respect to the maximum coefficients, the larger $g(m)$ in Eq.(5), the larger m becomes. This means that we should take only some least significant digits (LSDs) into considerration. The MILP algorithm applied only to 3-LSDs is described in [4]. However, the MILP algorithm classified in a complete search method (e.g. 0/1-variable method) requires a *long computation time*, which limits its usefulness to designing filters relatively. Generally, the number of conditions and the number of variables grow approximately as $O(2M^3)$, where M is the number of coefficient.

Unlike the MILP algorithm, the local search optimizes the coefficients set by changing always one coefficient in time to the next (higher or lower) closest quantized value. Moreover, it is achieved by evaluating the frequency response of the FIR filter with the resulting coefficients. The resulting number of conditions and the number of variables become $O(2M^2)$. Therefore, the optimization problem using local search is expected to have an M-fold speedup as compared with the MILP algorithm, except that the local search is inferior to the MILP algorithm in the light of its accuracy. In the next section, we propose a new local search algorithm based on the above concept.

3. Proposed algorithm

3.1 Local Search Optimization

The main objective in the optimization of the digital filters is usually to minimize the weighted peak error defined as

$$\varepsilon=\max_{0\leq\omega\leq\pi}W(\omega)\left|H(e^{j\omega})-H_d(e^{j\omega})\right|, \qquad (6)$$

where $H_d(e^{j\omega})$ and $H(e^{j\omega})$ represent the desired transfer function and the actual transfer function obtained after the completion the optimization, respectively. In addition, $W(\omega)$ represents a weighting function having different values in the passband and stopband region(s) of the FIR digital filter.

There are two main approaches available for the above optimization, namely the global and the local optimization. In the global optimization, the entire solution space is searched for the global minimum. On the other hand, in the local optimization, the solution space is searched in the neighborhood of a local minimum. In practice, if the local search is conducted around the global minimum, then the solution will coincide with that obtained from global search. The optimization of the FIR digital filters having CSD coefficients is achieved by using a combination of the global optimization and the local optimization in two stages as follows: In the first stage, the global optimization is carried out to minimize the weighted peak error

$$\varepsilon_1=\max_{0\leq\omega\leq\pi}W_1(\omega)\left|H_a(\omega)-H_d(\omega)\right| \qquad (7)$$

between an actual transfer function $H_a(\omega)$ and the desired transfer function $H_d(\omega)$ assuming that the actual coefficients. On the other hand, in the second stage, the local optimization is carried out to minimize the weighted peak error

$$\varepsilon_2=\max_{0\leq\omega\leq\pi}W_2(\omega)\left|H_{csd}(\omega)-H_a(\omega)\right| \qquad (8)$$

between the final transfer function $H_{csd}(\omega)$ and the transfer function $H_a(\omega)$ obtained in the first stage assuming that the CSD coefficients. Generally, the weighting function $W_1(\omega)$ in ε_1 and $W_2(\omega)$ in ε_2 are different in the passband and the stopband regions. In this way, the local optimization leads to a solution that approximately equals the result of the global optimization, but this optimization employs CSD (instead of actual) coefficients. This paper is concerned with the local optimization of FIR digital filters having CSD coefficients.

3.2 Proposed Local Search

The proposed local optimization for the FIR digital filters over the CSD coefficient space is facilitated by using a bivariate search as follows:

1. Calculate the actual coefficient values (h_a) associated with the transfer function $H_a(\omega)$ by using the Remez-exchange algorithm.
2. h_a are rounded to the closest CSD coefficients (h_{csd1}) as following

   $$h_{csd1}=Q_L\left[h_t\right], \qquad (9)$$

   where $Q_L\,[x]$ is L-digit CSD rounded value of x and x has the maximum number of SPT terms S_{max}.
3. Let $J(>0)$ represent the total number of distinguishable FIR digital filter coefficients. Here, J is

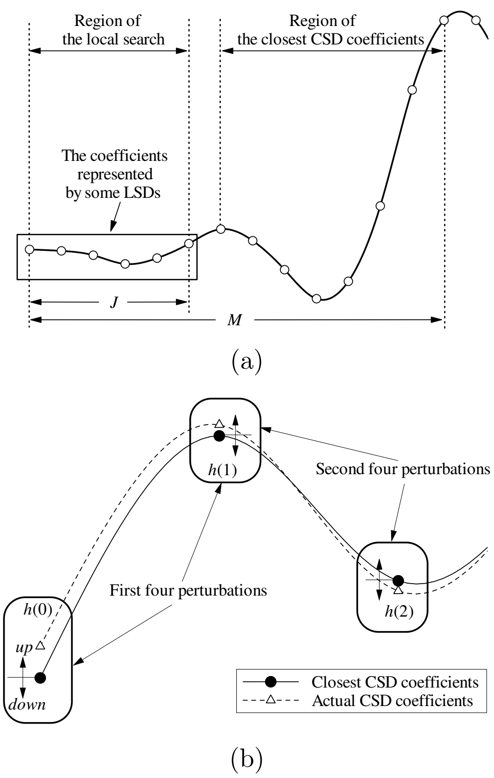


Figure 1. Conceptual diagram of the coefficient pair.

the partial number of the coefficients (i.e. number of coefficients represented by some LSDs only) in the symmetrical FIR digital filters. J is given by heuristic approach and indicates generally the number of coefficient represented within the LSDs range of 3 to 5 [4].

The coefficients are grouped into $J/2$ pairs. The total number of such coefficient pairs is

$$J + \frac{J(J-1)}{2} = \frac{J(J+1)}{2}, \qquad (10)$$

if the pairs are permitted to consist of nondistinct as well as distinct numbers.

4. Construct an echelon matrix $\boldsymbol{X}$ given by the following formula

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{h_{01}} & \boldsymbol{h_{02}} & \cdots & \boldsymbol{h_{0J-1}} & \boldsymbol{h_{0J}} \\ \boldsymbol{0} & \boldsymbol{h_{12}} & \cdots & \boldsymbol{h_{1J-1}} & \boldsymbol{h_{1J}} \\ \vdots & \ddots & & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{h_{J-1J}} \end{bmatrix}, \qquad (11)$$

$$(\boldsymbol{h_{01}}, \boldsymbol{h_{02}}, \cdots, \boldsymbol{h_{J-1J}} \neq \boldsymbol{0})$$

where $\boldsymbol{h_{J-1J}} = [h(J-1)\ h(J)]^T$ indicate distinct coefficient pair.

5. For each distinct coefficient pair $\boldsymbol{h_{J-1J}}$, four perturbations are performed by simultaneously rounding the first number in the pair up or down by one digit and by rounding the second number in the pair up or down by one digit. Similarly, for each nondistinct coefficient pair, two perturbations are performed by rounding the number up or down by one digit. Figure 1 shows conceptual diagram of four perturbations.

   The total of the perturbed coefficients is given by

$$2J + \frac{4J(J-1)}{2} = 2J^2. \qquad (12)$$

6. The local search is based on the iterative manner as described in the following: The unperturbed values of the coefficient pair are replaced with the perturbed values until no further reduction in ε_2 is achieved.
7. Check the stopping rule. If no further reduction in ε_2 is achieved, stop the process. Otherwise, go to step 5.

In the conventional local search algorithm, $2N^2$ pairs of the perturbed coefficients must be calculated. On the other hand, only $2J^2$ pairs of the perturbed coefficients are calculated in the proposed local search algorithm. It means that the proposed algorithm while requiring computational time is affected by appearance ratio of non-zero bits.

4. Design Examples

4.1 28th-order FIR Lowpass Filter

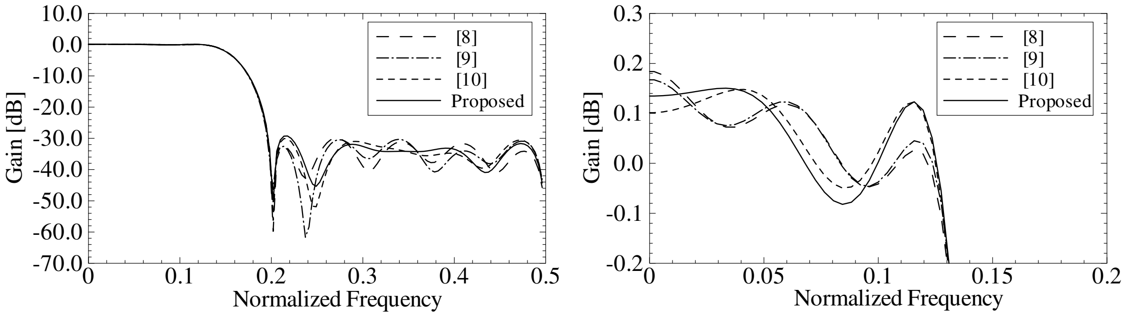
The 28th-order lowpass filter as described in [10] is considered. The design of lowpass filter whose normalized passband edge frequency (PF) is 0.1280, normalized stopband edge frequency (SF) is 0.2048, the maximum allowed error (or ripple) in passband (denoted as PR) is 0.0230 and the maximum allowed error (or ripple) in the stopband (denoted as SR) is 0.0320. In this example, the proposed algorithm is set to search only for the coefficients represented by 4-LSDs. The CSD coefficients obtained by using the proposed algorithm (with $S_{max} = 2$) and them expressed in [4], [8]–[10] are summarized in Table 1. These frequency response of the resulting filters are shown in Fig.2. The computational times required by a 1 GHz Pentium III processor (with 512 MB RAM running Windows98 and MATLAB 5.3) prepared to design the filter is also included in Table 1. From this table and Fig.2, it is found that the proposed algorithm can be used to design FIR filter which has small #SPT as the conventional algorithms. The filter response obtained by using this algorithm is much the same as obtained by using the other algorithms. In addition, the proposed local search algorithm performs about 2 times as fast as the conventional local search algorithms. This shortening of calculation time is very significant in the light of turn-around-time (TAT).

4.2 31st-order Hilbert Transformer

As the other example, the 31st-order Hilbert transformer [11] is designed by using the proposed method. The number of non-zero digit of the coefficients is given by 2-bits (i.e. $S_{max} = 2$) in this example. When the word length of the coefficients is more than 7-bit, we can design the 31st-order Hilbert transformer. Then,

Table 1. 28th-order lowpass filter coefficient values and filter responses obtained by different approaches.

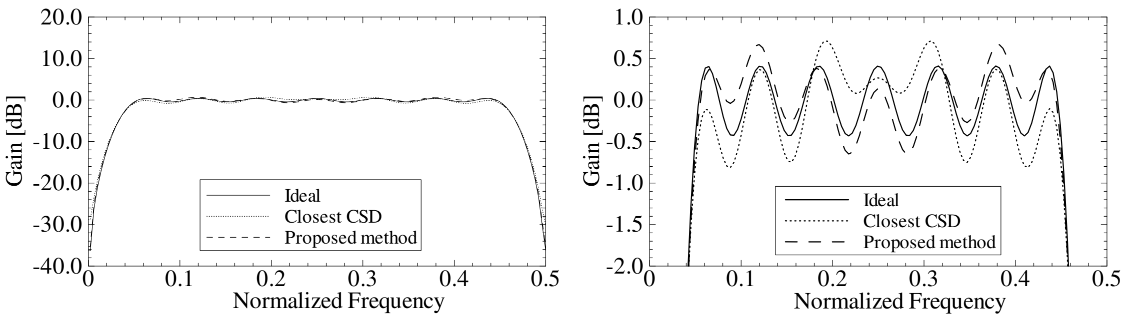
	Actual (h_a)	Closest CSD (h_{csd1})	MILP[4]	[8]	[9]	[10]	Proposed
$h(0)$	0.00681288	$+2^{-7}+2^{-10}$	$+2^{-8}+2^{-10}$	$+2^{-7}$	$+2^{-7}$	$+2^{-8}$	$+2^{-7}$
$h(1)$	0.00301232	$+2^{-8}-2^{-10}$	$+2^{-9}$	$+2^{-8}$	$+2^{-8}$	$+2^{-9}$	$+2^{-8}$
$h(2)$	− 0.00591135	$-2^{-7}+2^{-9}$	-2^{-8}	$-2^{-7}+2^{-9}$	$-2^{-7}+2^{-9}$	-2^{-8}	-2^{-8}
$h(3)$	− 0.01418709	$-2^{-6}+2^{-10}$	-2^{-6}	$-2^{-6}+2^{-10}$	-2^{-6}	-2^{-6}	-2^{-6}
$h(4)$	− 0.00919492	$-2^{-7}-2^{-10}$	-2^{-7}	-2^{-7}	-2^{-7}	-2^{-7}	-2^{-7}
$h(5)$	0.01100587	$+2^{-6}-2^{-8}$	$+2^{-6}-2^{-8}$	$+2^{-6}-2^{-8}$	$+2^{-6}-2^{-8}$	$+2^{-6}-2^{-8}$	$+2^{-6}-2^{-8}$
$h(6)$	0.02840078	$+2^{-5}-2^{-9}$	$+2^{-5}$	$+2^{-5}-2^{-9}$	$+2^{-5}$	$+2^{-5}$	$+2^{-5}-2^{-9}$
$h(7)$	0.01775964	$+2^{-6}+2^{-9}$	$+2^{-6}$	$+2^{-6}+2^{-9}$	$+2^{-6}$	$+2^{-6}$	$+2^{-6}+2^{-9}$
$h(8)$	− 0.02423816	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$
$h(9)$	− 0.06216134	-2^{-4}	-2^{-4}	-2^{-4}	-2^{-4}	-2^{-4}	-2^{-4}
$h(10)$	− 0.04196937	$-2^{-5}-2^{-7}$	$-2^{-5}-2^{-7}$	$-2^{-5}-2^{-7}$	$-2^{-5}-2^{-7}$	$-2^{-5}-2^{-7}$	$-2^{-5}-2^{-7}$
$h(11)$	0.06109336	$+2^{-4}$	$+2^{-4}$	$+2^{-4}$	$+2^{-4}$	$+2^{-4}$	$+2^{-4}$
$h(12)$	0.20879910	$+2^{-2}-2^{-5}$	$+2^{-2}-2^{-5}$	$+2^{-2}-2^{-5}$	$+2^{-2}-2^{-5}$	$+2^{-2}-2^{-5}$	$+2^{-2}-2^{-5}$
$h(13)$	0.31717250	$+2^{-2}+2^{-4}$	$+2^{-2}+2^{-4}$	$+2^{-2}+2^{-4}$	$+2^{-2}+2^{-4}$	$+2^{-2}+2^{-4}$	$+2^{-2}+2^{-4}$
	PF(0.1280)	0.1273	0.1281	0.1299	0.1309	0.1304	0.1280
	SF(0.2048)	0.1966	0.2001	0.1968	0.1958	0.1968	0.2019
	PR(0.0230)	0.0160	0.0151	0.0215	0.0195	0.0172	0.0182
	SR(0.0320)	0.0292	0.0301	0.0320	0.0305	0.0317	0.0320
	#SPT	26	20	23	20	19	21
	Computation time	———	320 s	51.0 s	51.2 s	93.1 s	32.0 s



(a) Frequency response.

(b) Frequency response (enlarged view).

Figure 2. Frequency response of 28th-order lowpass filter.



(a) Frequency response.

(b) Frequency response (enlarged view).

Figure 3. Frequency response of 31st-order Hilbert transformer.

the word length of the coefficients of the Hilbert transformer is set to 8-bits. In this example, the proposed algorithm is set to search only for coefficient represented by 4-LSDs.

Table 2 indicates that required #SPT of 31st-order Hilbert transformer is decreased from 13 to 10. The frequency response of this filter is illustrated in Fig.3. From this figure, it is shown that SR of the proposed method is extremely improved with respect to that of the closest CSD.

5. Implementation Results

A testbed prepared for evaluating the above examples is constructed with Xilinx XC2S30-6 (SpartanII series)

Table 3. Comparison of mapping results for 28th-order FIR lowpass filter.

	Closest CSD	MILP[4]	[8]	[9]	[10]	Proposed
Adders/Subtracters	33	29	31	29	29	31
Slices	302/432 (69.9%)	282/432 (65.3%)	300/432 (69.4%)	282/432 (65.3%)	282/432 (65.3%)	282/432 (65.3%)
A (4-input LUTs)	451/864 (52.2%)	502/864 (58.1%)	419/864 (48.5%)	417/864 (48.3%)	428/864 (49.5%)	448/864 (51.9%)
T (Time)	15.0 ns	14.1 ns	15.9 ns	13.6 ns	13.2 ns	12.3 ns
A·T (Area-Time product)	1.00	1.05	0.98	0.83	0.84	0.81

Table 4. Comparison of mapping results for 31st-order Hilbert transformer.

	Actual (2's comp. bits)	Closest CSD ($S_{max}=2$, CSD bits)	Final CSD ($S_{max}=2$, CSD bits)
Adders/Subtracters	56	18	17
Slices	247/432 (57.2%)	82/432 (18.9%)	79/432 (18.3%)
A (4-input LUTs)	366/864 (42.4%)	120/864 (13.9%)	113/864 (13.1%)
T (Time)	17.3 ns	11.3 ns	8.22 ns
A·T (Area-Time product)	1.00	0.21	0.15

Table 2. 31st-order Hilbert transformer coefficient values and filter responses.

	Actual (h_a)	Closest CSD (h_{csd1})	Final CSD (h_{csd})
$h(0)$	-0.03413334	-2^{-5}	-2^{-5}
$h(1)$	0.00000000	0	0
$h(2)$	-0.02044246	$-2^{-5}+2^{-7}$	-2^{-6}
$h(3)$	0.00000000	0	0
$h(4)$	-0.01794832	-2^{-6}	-2^{-6}
$h(5)$	0.00000000	0	0
$h(6)$	-0.00587839	-2^{-6}	-2^{-7}
$h(7)$	0.00000000	0	0
$h(8)$	-0.02052302	$+2^{-5}-2^{-7}$	-2^{-6}
$h(9)$	0.00000000	0	0
$h(10)$	0.07036104	$+2^{-4}+2^{-7}$	$+2^{-4}$
$h(11)$	0.00000000	0	0
$h(12)$	0.17509460	$+2^{-3}+2^{-5}$	$+2^{-3}+2^{-5}$
$h(13)$	0.00000000	0	0
$h(14)$	0.62373170	$+2^{-1}+2^{-3}$	$+2^{-1}+2^{-3}$
$h(15)$	0.00000000	0	0
PF(0.0500, 0.4500)		0.0550, 0.04450	0.0549, 0.4451
SF(0.0400, 0.4600)		0.0390, 0.46080	0.0392, 0.4608
PR(0.1222)		0.1842	0.1602
SR(0.0153)		0.0181	0.0173
#SPT		13	10

FPGA. The Xilinx XC2S30-6 FPGA consists of a 12×18 array of CLBs, each having two 4-input LUTs, one 3-input LUT and two D flip-flops (D-FFs). Two examples are described by VHDL for implementation, and also are actually implemented on XC2S30-6 using Leonard Spectrum v2001.1d (Exemplar) and ISE Alliance (Xilinx). In those implementations, the automatic placement and routing compilation option are used.

The proposed algorithm also enables the use of the common subexpression reduction algorithms (see, e.g. [12]– [15]) for the evaluation of the filter costs. In those examples, the subexpression reduction algorithm based on the method described in [15] is carried out for all the coefficients satisfying the amplitude specifications.

To evaluate the FIR filters we introduce an area-time product (A·T) which is motivated by practical cost measures for circuits. Comparisons are done using three metrics: A=area, T=time, and A·T=area×time, where T refers to the data sampling period or the inverse of the throughput. Area is measured by the number of required 4-input LUTs.

5.1 28th-order FIR Lowpass Filter

Synthesis results in terms of module count and speed are summarized in Table 3. As you can see, the proposed filter is the fastest speed of all the local search algorithms, but the increase in the number of gate cost is very small. The implementation example shows that the area-time product of the proposed 28th-order lowpass filter is about 2–17% smaller than those of the other local search algorithms.

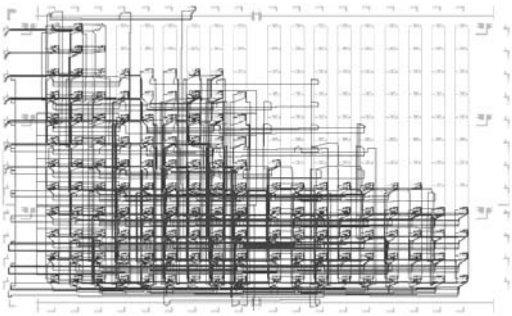
5.2 31st-order Hilbert Transformer

Synthesis results in terms of module count and speed are summarized in Table 4. Figure 4 illustrates mapping results of the logic in the FPGA chip. The experimental results show that the proposed Hilbert transformer is superior to the others in all parameters. We find that area-time product of the proposed Hilbert transformer is about 85% smaller than that of a 2's complement implementation.

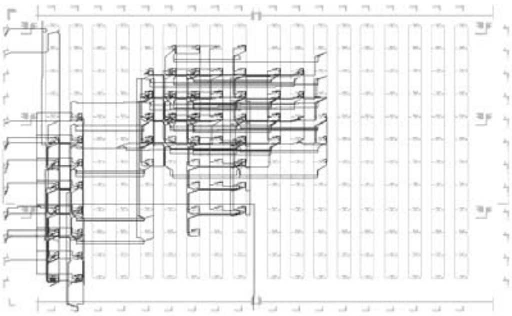
6. Conclusions

In this paper, a new design of the optimization of the FIR digital filters over the Canonic Signed-Digit (CSD) coefficient space has been presented. This algorithm has been about 2 times as fast as the conventional local search algorithms. The usefulness of this algorithm has been shown through two examples and two FPGA implementations.

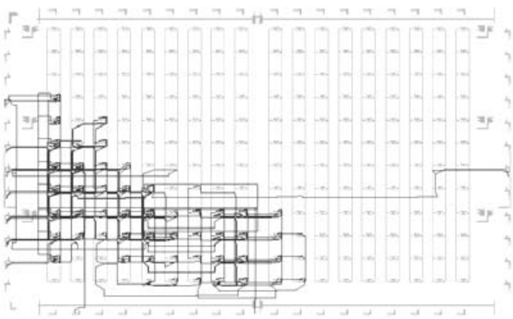
The scope of this algorithm in the design of the multiplierless FIR filters (e.g. mutilate filters, 2-D filters)



(a) 2's complement.



(b) Closest CSD.



(c) Final CSD.

Figure 4. Mapping results of 31st-order Hilbert transformer.

has a substantial potential. Extensions of the procedure to deal with SPT terms are undertaken. Further investigation is required to assess the performance of this algorithm for the design of multiplierless IIR filters.

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