Grasp Stability Analysis of Two Objects with both Friction and Frictionless Contacts in Two Dimensions

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Abstract:
This paper analyzes grasp stability of two objects with both friction and frictionless contacts in two dimensions. In the grasp of two objects, there exist multiple contact points which mean one contact point between two objects and plural contact points between objects and fingers. In the previous work, all these contact points are assumed to be either only with friction or only without friction. This paper discusses more practical grasp, in which there exist both friction contacts and frictionless contacts. In this case, both rolling motions and sliding motions occur due to friction contact and frictionless contact in the grasp system simultaneously. The grasp stability is evaluated by the potential energy stored in the grasp system. By using numerical examples, effect of friction is demonstrated.

1. Introduction

Multifingered robot hands have potential ability of fine and dexterous manipulation. In order to realize these excellent technologies, many works [1]-[10] have explored grasp and manipulation by the robot hand. This paper discusses grasp stability of multiple objects.

Stability is the tendency of a system to return to an equilibrium state when the system is displaced from this state. It is required for the hand to maintain the grasp stable against external disturbances. Nguyen [3] analyzed grasp stability by replacing a finger with a virtual spring model. The stability is evaluated by positive definiteness of a hessian of potential energy stored in the grasp system. The hessian is called a grasp stiffness matrix. Curvature effect [2,4,7] and dynamic stability [5,6] were also discussed. For efficiency of tasks, multiple objects had better been grasped and manipulated by a multifingered robot hand [1,9,10]. Yamada et al. [8] investigated grasp stability of two objects. In this work, however, all these contact points are assumed to be either only with friction or only without friction.

In practical grasps, there exists the case where large-friction objects and small-friction objects are grasped by a multifingered hand, and also exists the case where surface of each object consists of both friction and frictionless regions. Therefore, both rolling motions and sliding motions occur due to friction contact and frictionless contact in the grasp system simultaneously.

This paper discusses these practical grasps, in which there exist both friction and frictionless contacts. It is shown that the number of independent parameters of objects' motion becomes 5 in case of sliding contact and 4 in case of rolling contact. Displacement of fingertip position is derived. The relationship between finger displacement and reaction force is replaced with a 2-dimensional spring model. The potential energy of the grasp system is derived and the hessian of the energy is provided. The grasp stability is evaluated by the hessian. From this analysis, the effect of friction, fingertip position, fingertip force, and shape of object and finger is clarified. By using numerical examples, effect of friction is demonstrated.

2. Problem Formulation

As shown in Figure 1, we suppose that a multifingered robot hand grasps two objects in two dimensions.

2.1 Assumptions

Stability of the grasp system is analyzed with the following assumptions. 1) Both object and fingertip are rigid. 2) The contact between two bodies is of single contact. 3) Contact position, normal direction, and local curvature at the contact point are known. 4) Initial configuration is in

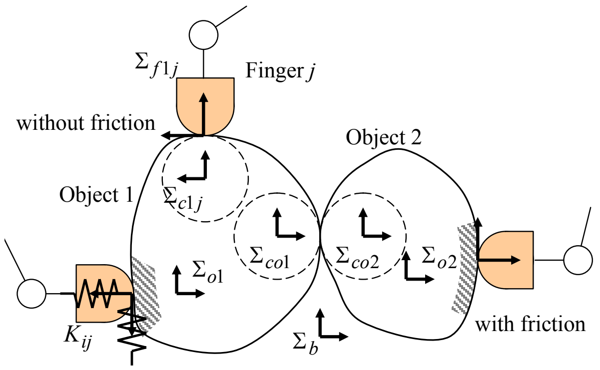


Figure 1: Two objects grasped by a multi-fingered hand.

equilibrium. 5) Infinitesimal displacement of object occurs due to external disturbances. 6) No rotation of fingertip occurs. 7) The relationship between displacement of fingertip position and reaction force can be replaced with a two-dimensional spring model.

2.2 Symbols

The following coordinate frames shown in Figure 1 are utilized. Σ_b is base frame, Σ_{oi} object frame fixed on the i-th object, Σ_c contact frame fixed at contact point between grasped objects, Σ_{coi} contact frame fixed at the center of the approximated circle at contact point between objects, Σ_{cij} contact frame on the i-th object with the j-th finger, Σ_{fij} contact frame on the j-th finger with i-th object, respectively.

Contact position, normal direction, and local curvature at the contact point are denoted as follows: ${}^{oi}\boldsymbol{c}_{coi}$ is contact position on the i-th object with the other object, ${}^{oi}\boldsymbol{c}_{cij}$ contact position on the i-th object with the j-th finger, ${}^{oi}\boldsymbol{n}_{coi}$ outward normal direction at ${}^{oi}\boldsymbol{c}_{coi}$, ${}^{oi}\boldsymbol{n}_{cij}$ outward normal direction at ${}^{oi}\boldsymbol{c}_{cij}$, κ_{coi} local curvature of the i-th object at ${}^{oi}\boldsymbol{c}_{coi}$, κ_{cij} local curvature of the i-th object at ${}^{oi}\boldsymbol{c}_{cij}$, κ_{fij} local curvature of the j-th finger at the contact, respectively. The left and top subscript means reference coordinate frame.

Position and orientation of the frames are denoted by the following homogeneous matrices.

$$ {}^{oi}T_{coi} = \left[\begin{array}{c|c} {}^{oi}R_{coi} & {}^{oi}\boldsymbol{p}_{coi} \\ \hline 0 & 1 \end{array}\right], $$

$$ {}^{oi}T_{cij} = \left[\begin{array}{c|c} {}^{oi}R_{cij} & {}^{oi}\boldsymbol{p}_{cij} \\ \hline 0 & 1 \end{array}\right], \quad {}^{cij}T_{fij} = \left[\begin{array}{c|c} I_2 & {}^{cij}\boldsymbol{p}_{fij} \\ \hline 0 & 1 \end{array}\right], \tag{1} $$

where the position $\boldsymbol{p}$ and the rotation R are calculated by using the contact position $\boldsymbol{c}$, the normal direction $\boldsymbol{n}$, and the curvature κ at the contact point.

$$ {}^{oi}R_{cij} = [{}^{oi}\boldsymbol{n}_{cij}, {}^{oi}\boldsymbol{t}_{cij}], \quad {}^{oi}R_{coi} = \gamma_i [{}^{oi}\boldsymbol{n}_{coi}, {}^{oi}\boldsymbol{t}_{coi}], $$

$$ {}^{oi}\boldsymbol{p}_{cij} = {}^{oi}\boldsymbol{c}_{cij} - \kappa_{cij}^{-1}\{{}^{oi}\boldsymbol{n}_{cij}\}, \quad {}^{oi}\boldsymbol{p}_{coi} = {}^{oi}\boldsymbol{c}_{coi} - \kappa_{coi}^{-1}\{{}^{oi}\boldsymbol{n}_{coi}\}, $$

$$ {}^{cij}\boldsymbol{p}_{fij} = \tilde{\kappa}_{ij}^{-1}\boldsymbol{u}_1, \quad \tilde{\kappa}_{ij}^{-1} = \kappa_{cij}^{-1} + \kappa_{fij}^{-1}, \quad \boldsymbol{t} = \mathrm{Rot}(\pi/2)\boldsymbol{n}, $$

$$ \boldsymbol{u}_1 = [1,0]^T, \quad \boldsymbol{u}_2 = [0,1]^T, \quad \gamma_1 = 1, \quad \gamma_2 = -1. \tag{2} $$

Displacement of the coordinate frame is denoted as follows: $\delta X_{oi} = [\delta x_{oi}, \delta y_{oi}, \delta\zeta_{oi}]^T$ is displacement of Σ_{oi} with respect to the initial Σ_{oi}, $\delta X_{coi} = [\delta x_{coi}, \delta y_{coi}, \delta\zeta_{coi}]^T$ displacement of Σ_{coi} with respect to the initial Σ_{coi}, $\delta X_{cij} = [\delta x_{cij}, \delta y_{cij}, \delta\zeta_{cij}]^T$ displacement of Σ_{cij} with respect to the initial Σ_{cij}, $\delta\boldsymbol{x}_{fij} = [\delta x_{fij}, \delta y_{fij}]^T$ displacement of Σ_{fij} with respect to the initial Σ_{fij}, respectively. The homogeneous matrices corresponding to these displacements are denoted by

$$ {}^{oi}T_{oi'}(\delta X_{oi}) = \left[\begin{array}{c|c} \mathrm{Rot}(\delta\zeta_{oi}) & \delta\boldsymbol{x}_{oi} \\ \hline 0 & 1 \end{array}\right], $$

$$ {}^{coi}T_{coi'}(\delta X_{coi}) = \left[\begin{array}{c|c} \mathrm{Rot}(\delta\zeta_{coi}) & \delta\boldsymbol{x}_{coi} \\ \hline 0 & 1 \end{array}\right], $$

$$ {}^{cij}T_{cij'}(\delta X_{cij}) = \left[\begin{array}{c|c} \mathrm{Rot}(\delta\zeta_{cij}) & \delta\boldsymbol{x}_{cij} \\ \hline 0 & 1 \end{array}\right], $$

$$ {}^{fij}T_{fij'}(\delta\boldsymbol{x}_{fij}) = \left[\begin{array}{c|c} I_2 & \delta\boldsymbol{x}_{fij} \\ \hline 0 & 1 \end{array}\right], \tag{3} $$

where

$$ \delta\boldsymbol{x} = [\delta x, \delta y]^T, \quad \mathrm{Rot}(\bullet) = \begin{bmatrix} \cos(\bullet) & -\sin(\bullet) \\ \sin(\bullet) & \cos(\bullet) \end{bmatrix} \tag{4} $$

and the symbol ' means after the displacement. From Assumption 1), ${}^{oi'}T_{coi'} = {}^{oi}T_{coi}$ and ${}^{oi'}T_{cij'} = {}^{oi}T_{cij}$.

The spring stiffness described in Assumption 7) is denoted by $K_{ij} = \mathrm{diag}[k_{xij}, k_{yij}]$ and is fixed along Σ_{fij}, where $k_{xi} > 0$ and $k_{yi} > 0$. The spring is compressed at the initial configuration and generates initial fingertip force $\boldsymbol{f}_{ij}$. The amount of the spring compression is denoted by $\delta\boldsymbol{x}_{fij0} = [\delta x_{fij0}, \delta y_{fij0}]^T$. The fingertip force is represented as

$$ \boldsymbol{f}_{ij} = K_i \delta\boldsymbol{x}_{fij0}. \tag{5} $$

3. Position Displacement

In this section, we derive displacement of object position and finger position.

3.1 Displacement of the frame Σ_{coi}

The displacement of object position can be represented by five parameters shown in Figure 2. α_1 and α_2 mean translation of Σ_c. α_3 stands for rotation. These three parameters are the same as the displacement of single object with respect to Σ_c. α_4 and α_5 mean arc length of rolling or sliding. The displacement of Σ_{coi}, (i=1,2) is represented by

$$ \delta\boldsymbol{x}_{coi} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} - \{\mathrm{Rot}(\alpha_3) - I_2\}{}^{coi}\boldsymbol{c}_{coi}, \tag{6.a} $$

$$ \delta\zeta_{coi} = \alpha_3 + \kappa_{coi}\left(\frac{1+\gamma_i}{2}\alpha_4 + \frac{1-\gamma_i}{2}\alpha_5\right), \tag{6.b} $$

where

$$ {}^{coi}\boldsymbol{c}_{coi} = \gamma_i \kappa_{coi}^{-1}\boldsymbol{u}_1, \tag{7} $$

The fractions in (6.b) become either 0 or 1 for selection from α_4 and α_5.

3.2 Displacement of the frames Σ_{oi} and Σ_{cij}

The relation between Σ_{oi} and Σ_{coi} is given by

$$\{{}^{oi}T_{oi'}(\delta X_{oi})\}\{{}^{oi'}T_{coi'}\} = \{{}^{oi}T_{coi}\}\{{}^{coi}T_{coi'}(\delta X_{coi})\}. \tag{8}$$

and the relation between Σ_{oi} and Σ_{cij} is given by

$$\{{}^{oi}T_{oi'}(\delta X_{oi})\}\{{}^{oi'}T_{cij'}\} = \{{}^{oi}T_{cij}\}\{{}^{cij}T_{cij'}(\delta X_{cij})\}. \tag{9}$$

Then we have

$${}^{cij}T_{cij'}(\delta X_{cij}) = \{{}^{cij}T_{coi}\}\{{}^{coi}T_{coi'}(\delta X_{coi})\}\{{}^{cij'}T_{coi'}\}^{-1}. \tag{10}$$

The displacement of Σ_{cij} is given by

$$\delta\boldsymbol{x}_{cij} = {}^{coi}R_{cij}^{T}[\delta\boldsymbol{x}_{coi} + \{\mathrm{Rot}(\delta\zeta_{coi}) - I_2\}{}^{coi}\boldsymbol{p}_{cij}], \tag{11.a}$$

$$\delta\zeta_{cij} = \delta\zeta_{coi}. \tag{11.b}$$

where

$${}^{coi}\boldsymbol{p}_{cij} = \{{}^{coi}R_{oi}\}\{{}^{oi}\boldsymbol{p}_{cij} - {}^{oi}\boldsymbol{p}_{coi}\}. \tag{12}$$

3.3 Displacement of the frame Σ_{fij}

Each finger shifts on the object surface as shown in Figure 3. The direction of the contact position on the finger surface is denoted by β_{ij}. From Assumption 6), the displacement of Σ_{fij} is given by

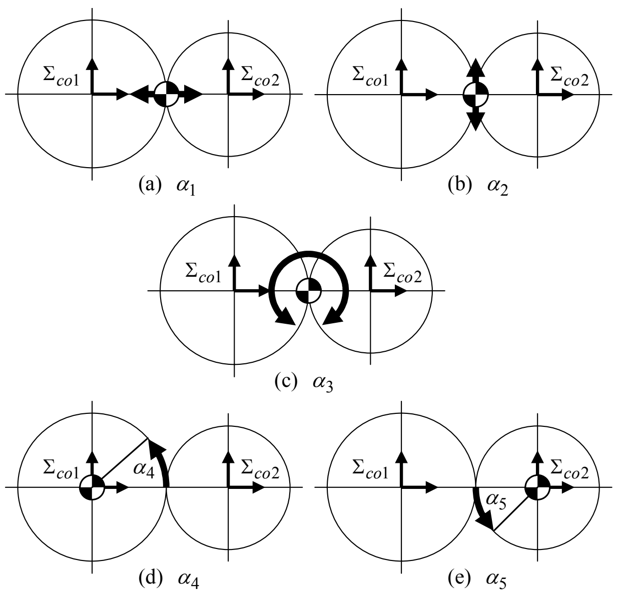


Figure 2: Five types of object motion.

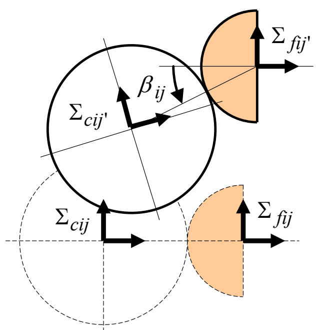


Figure 3: Displacement of finger position

$$\delta\boldsymbol{x}_{fij}(\boldsymbol{\alpha},\beta_{ij}) = \delta\boldsymbol{x}_{cij}(\boldsymbol{\alpha}) + \{\mathrm{Rot}(\beta_{ij}) - I_2\}{}^{cij}\boldsymbol{p}_{fij}. \tag{13}$$

The parameters $\boldsymbol{\alpha}$ and β_{ij} depend on whether friction exists or not.

4. Grasp Stability

The potential energy stored in the grasp system is given by

$$U = \sum_{i,j} U_{ij}(\boldsymbol{\alpha},\beta_{ij}), \tag{14}$$

where U_{ij} is the potential energy stored in the j-th finger contacting with the i-th object:

$$\begin{aligned} &U_{ij}(\boldsymbol{\alpha},\beta_{ij}) \\ &= \frac{1}{2}\{\delta\boldsymbol{x}_{fij0} + \delta\boldsymbol{x}_{fij}(\boldsymbol{\alpha},\beta_{ij})\}^T K_{ij}\{\delta\boldsymbol{x}_{fij0} + \delta\boldsymbol{x}_{fij}(\boldsymbol{\alpha},\beta_{ij})\} \end{aligned}. \tag{15}$$

From Appendix, the grasp stability is analyzed by the hessian of the potential energy. In the following subsections, the parameter $\boldsymbol{\varepsilon}$ is clarified and the constraint of β_{ij} is derived. Then the hessian is provided.

4.1 Frictionless contact between two objects

If the contact between two objects is without friction, the displacement parameters of the system are represented by

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{os} := [\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5]^T \in \Re^5. \tag{16}$$

4.1.1 Frictionless contact between finger and object

If the contact between finger and object is without friction, each finger slides on the object. Since the potential energy U_{ij} must be local minimum, β_{ij} is constrained by

$$\partial U_{ij}(\boldsymbol{\alpha},\beta_{ij})/\partial\beta_{ij} = 0, \tag{17.a}$$

$$\partial^2 U_{ij}(\boldsymbol{\alpha},\beta_{ij})/\partial\beta_{ij}^2 > 0. \tag{17.b}$$

From (6), (11), (13), (15), (16), and (17.a), we have

$$H_{ij}^{fs} = s_{aij}^{fs}\boldsymbol{v}_{ai}\boldsymbol{v}_{ai}^T + s_{bij}^{fs}\boldsymbol{v}_b\boldsymbol{v}_b^T + E_{coi}W_{ij}^{fs}K_{ij}^{fs}(W_{ij}^{fs}E_{coi})^T, \tag{18}$$

where

$$s_{aij}^{fs} := -\{{}^{coi}\boldsymbol{p}_{cij}\}^T\{{}^{coi}\boldsymbol{f}_{ij}\},\ \ s_{bij}^{fs} := \{{}^{coi}\boldsymbol{c}_{coi}\}^T\{{}^{coi}\boldsymbol{f}_{ij}\},$$

$$\boldsymbol{v}_{ai} := E_{coi}[0,0,1]^T,\ \ \boldsymbol{v}_b := [0,0,1,0,0]^T,$$

$$E_{coi} := \left[\begin{array}{c|c} I_2 & 0_2 \\ \hline -[{}^{coi}\boldsymbol{c}_{coi}\otimes] & 1 \\ \hline 0\ \ 0 & (1+\gamma_i)\kappa_{coi}/2 \\ 0\ \ 0 & (1-\gamma_i)\kappa_{coi}/2 \end{array}\right],$$

$$W_{ij}^{fs} := \begin{bmatrix} I_2 \\ [{}^{coi}\boldsymbol{p}_{cij}\otimes] \end{bmatrix}\{{}^{coi}R_{cij}\},\ \ {}^{coi}\boldsymbol{f}_{ij} := {}^{coi}R_{cij}\boldsymbol{f}_{ij},$$

$$K_{ij}^{fs} = \mathrm{diag}[k_{xij}, -\frac{\tilde{\kappa}_{ij}f_{xij}k_{yij}}{k_{yij} - \tilde{\kappa}_{ij}f_{xij}}],$$

$$[\begin{bmatrix} p_x \\ p_y \end{bmatrix}\otimes] = [-p_y, p_x] \,. \tag{19}$$

The superscript "fs" means sliding motion of finger due to friction contact. From (17.b), k_{yij} and f_{xij} must satisfy

$$k_{yij} - \tilde{\kappa}_{ij} f_{xij} > 0 \,. \tag{20}$$

Note that H_{ij}^{fs} of (18) is the same as H_{ij}^{s} of Ref. [8].

4.1.2 Friction contact between finger and object

If the contact between finger and object is with friction, each finger rolls on the object. From Assumption 6), β_{ij} is given by

$$\beta_{ij} = \tilde{\kappa}_{ij} \kappa_{cij}^{-1} \zeta_{coi} \,. \tag{21}$$

Then we have

$$H_{ij}^{fr} = s_{aij}^{fr} \boldsymbol{v}_{ai} \boldsymbol{v}_{ai}^T + s_{bij}^{fr} \boldsymbol{v}_b \boldsymbol{v}_b^T + E_{coi} W_{ij}^{fr} K_{ij} (E_{coi} W_{ij}^{fr})^T \tag{22}$$

where

$$s_{aij}^{fr} := s_{aij}^{fs} - \tilde{\kappa}_{ij} \kappa_{cij}^{-2} \{{}^{coi}R_{cij} \boldsymbol{u}_1\} \{{}^{coi}\boldsymbol{f}_{ij}\} \,, \quad s_{bij}^{fr} := s_{bij}^{fs} \,,$$

$$W_{ij}^{fr} := W_{ij}^{fs} + \begin{bmatrix} 0_{2\times 2} \\ [(\kappa_{cij}^{-1} \boldsymbol{u}_1)\otimes] \end{bmatrix} , \tag{23}$$

The superscript "fr" means rolling motion of finger due to friction contact. Note that the size of row and column of H_{ij}^{fr} is different from that of H_{ij}^{r} of Ref. [8], because of frictionless contact between two objects.

4.1.3 Hessian of the grasp

In case of frictionless contact between two objects, the hessian of the grasp system is given by

$$H^{os} = \sum_{i,j} H_{ij}^{*} \in \Re^{5\times 5} \,. \tag{24}$$

The superscript "os" means sliding motion between two objects due to frictionless contact. The symbol * stands for "fs" or "fr".

The hessian H^{os} depends on presence of friction between finger and object, contact position, contact normal, curvature at contact point, contact force, and spring stiffness. The grasp stability is evaluated by using positive definiteness of H^{os}.

4.1.4 Difference between H_{ij}^{fr} and H_{ij}^{fs}

The effect of friction between finger and object is obtained by the difference between H_{ij}^{fr} and H_{ij}^{fs}:

$$\begin{aligned} H_{ij}^{fd} &:= H_{ij}^{fr} - H_{ij}^{fs} \\ &= \frac{{k_{yij}}^2}{k_{yij} - \tilde{\kappa}_{ij} f_{xij}} E_{coi} \{\boldsymbol{w}_{ij}^{fd}\} \{\boldsymbol{w}_{ij}^{fd}\}^T E_{coi}^T \,, \end{aligned} \tag{25}$$

where

$$\boldsymbol{w}_{ij}^{fd} = W_{ij}^{fs} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{\kappa_{cij}^{-1}(k_{yij} - \tilde{\kappa}_{ij} f_{xij})}{k_{yij}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \in \Re^3 \,. \tag{26}$$

It is clarified that friction between finger and object makes grasps stable higher because (25) is positive semi-definite.

4.2 Friction contact between two objects

If the contact between two objects is with friction, α is constrained by

$$\alpha_5 = -\alpha_4 \,. \tag{27}$$

Then we have

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{or} := [\alpha_1, \alpha_2, \alpha_3, \alpha_4]^T \in \Re^4 \,. \tag{28}$$

The hessian is given by

$$H^{or} = A(\sum_{i,j} H_{ij}^{*}) A^T \in \Re^{4\times 4} \,, \tag{29}$$

where

$$A := \left[\begin{array}{c|cc} I_3 & \multicolumn{2}{c}{0_{3\times 2}} \\ \hline 0_{1\times 3} & 1 & -1 \end{array}\right] \in \Re^{4\times 5} \,. \tag{30}$$

The superscript "or" means rolling contact between two objects.

5. Numerical Examples

We evaluate the grasp stability by using the derived hessian of (24) and (29). A 4-fingered grasp shown in Figure 4 is considered. The bold line stands for two grasped objects, which are the same shape. The left object is numbered the 1st object, and the right one is numbered the 2nd object. The frame Σ_{oi} is fixed at the center of the i-th object. The contact positions on the object are on the virtual circle with radius r. The curvature of the object at contact point between two objects is assigned with $\kappa_{coi} = \kappa_{co}$. The contact position with respect to Σ_{o1} and Σ_{o2} are respectively given by

$${}^{o1}\boldsymbol{c}_{co1} = [r, 0]^T \,, \quad {}^{o2}\boldsymbol{c}_{co2} = [-r, 0]^T \,, \tag{31}$$

The j-th finger makes contact with the i-th object at

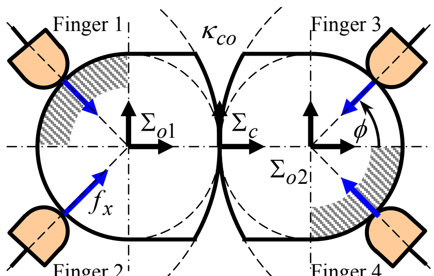


Figure 4: Numerical Example

$$ {}^{oi}\boldsymbol{c}_{cij} = r[\cos\phi_{ij}, \sin\phi_{ij}]^T , \tag{32} $$

where ϕ_{ij} means position parameter.

$$ \phi_{11} = \pi - \phi ,\ \phi_{12} = \pi + \phi ,\ \phi_{23} = \phi ,\ \phi_{24} = -\phi \tag{33} $$

Each finger is assigned with the same curvature, i.e., $\kappa_{fij} = \kappa_f$. The normal and the tangent components of spring stiffness are assigned with the same value, i.e., $k_{xij} = k_{yij} = k$. Initial fingertip force with respect to Σ_{fij} is assigned with

$$ \boldsymbol{f}_{ij} = [f_x, 0]^T . \tag{34} $$

6 cases of contact friction described in Table 1 are investigated. In the first 3 cases, 5 eigenvalues are obtained from the hessian because of frictionless contact between two objects. In the last 3 cases, 4 eigenvalues are obtained because of friction contact between two objects. Note that Case 1 and 6 are treated in Ref. [8].

The above grasp parameters are assigned with the following values:

$$ r = 0.02(\mathrm{m}) ,\ \phi = \pi/3(\mathrm{rad}) ,\ \kappa_f = 200(\mathrm{m}^{-1}) , $$
$$ k = 500(\mathrm{N/m}) ,\ f_x = 3(\mathrm{N}) . \tag{35} $$

The eigenvalues related with κ_{co} are depicted in Figure 5 - 10. Small eigenvalues are shown in Figure (b) in which vertical axis is magnified around zero. In Case 1, 2, 4, and 5, the grasp becomes unstable around $\kappa_{co} > 50$. These 4 cases include frictionless contact between finger and object. By comparing the smallest eigenvalues, it can be seen that the grasp with friction between two objects is more stable than that without friction.

6. Conclusions

The grasp stability of two objects with both friction and frictionless contacts in two dimensions was analyzed. The hessian of the potential energy stored in the grasp system was provided. The grasp stability was evaluated by the eigenvalues of the hessian. By using numerical examples, stability of practical grasps with both friction and frictionless contacts were evaluated. Our results can be applied to a design problem of the grasp parameters for making stable grasp.

References

[1] K. Harada, M. Kaneko, and Tsuji, "Rolling Based Manipulation for Multiple Objects," *Proc. of Int. Conf. on Robotics and Automation*, pp. 3888-3895, 2000.

[2] W. S. Howard and V. Kumar, "On the Stability of Grasped Objects," *IEEE Trans. on Robotics and Automation*, Vol. 12, No. 6, pp. 904-917, 1996.

[3] V. D. Nguyen, "Constructing Stable Grasps," *Int. Journal of Robotics Research*, Vol. 7, No. 3, pp. 3-16, 1988.

[4] E. Rimon and J. W. Burdick, "Mobility of bodies in contact - Part II: How Forces are Generated by Curvature Effects," *IEEE Trans. of Robotics and Automation*, pp. 709-717, 1998.

[5] K. B. Shimoga, "Robot grasp synthesis algorithms: A survey." *Int. Journal of Robotics Research*, Vol. 15, No. 3, pp. 230-266, 1996.

[6] C. H. Xiong, Y. F. Li, H. Ding, and Y. L. Xiong, "On the Dynamic Stability of Grasping," *Int. Journal of Robotics Research*, Vol. 18, No. 9, pp. 951-958, 1999.

[7] T. Yamada, T. Koishikura, Y. Mizuno, N. Mimura, and Y. Funahashi, "Stability Analysis of 3D Grasps by A Multifingered Hand," *Proc. of Int. Conf. on Robotics and Automation*, pp. 2466-2473, 2001.

[8] T. Yamada, T. Ooba, T. Yamamoto, N. Mimura, and Y. Funahashi, "Grasp Stability Analysis of Two Objects in Two Dimensions," *Proc. of Int. Conf. on Robotics and Automation*, pp. 772-777, 2005.

[9] T. Yoshikawa, T. Watanabe, and M. Daito, "Optimization of Power Grasps for Multiple Objects," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 1786-1791, 2001.

[10] Y. Yu, K. Fukuda, and S. Tsuji, "On Computation of Grasp Internal Forces for Stably Grasping," *Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 1776-1781, Hawaii, 2001.

Appendix: Definition of Grasp Stability

The potential energy stored in the grasp system is calculated by

$$ U(\varepsilon) = \sum_{i,j} U_{ij}(\varepsilon) , \tag{36} $$

where ε means independent parameters of objects' displacement. Performing Taylor's expansion, the function $U(\varepsilon)$ can be written as

$$ U(\varepsilon) = U(0) + \varepsilon^T G + \frac{1}{2}\varepsilon^T H \varepsilon + ... \tag{37} $$

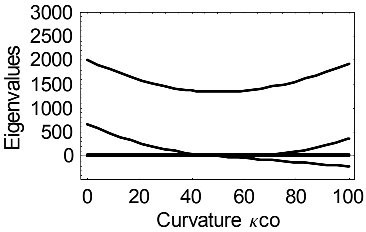
where G and H are the gradient and the hessian, respectively. The grasp is stable if the following two conditions are satisfied.

1) $G = 0$
2) H is positive definite.

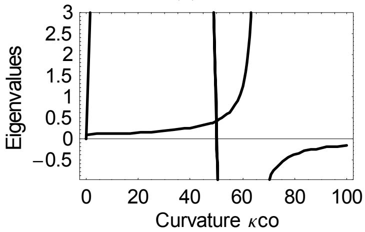
Condition 1) is satisfied by Assumption 4). Hence, the grasp stability can be evaluated by Condition 2).

Table 1: Condition of contact friction (w/: with friction, w/o: without friction)

Case	between two objects	between object and				Hessian of the grasp	Figure No.
		Finger 1	Finger 2	Finger 3	Finger 4		
1	w/o	w/o	w/o	w/o	w/o	$H_{11}^{fs}+H_{12}^{fs}+H_{23}^{fs}+H_{24}^{fs}$	5
2	w/o	w/	w/o	w/o	w/	$H_{11}^{fr}+H_{12}^{fs}+H_{23}^{fs}+H_{24}^{fr}$	6
3	w/o	w/	w/	w/	w/	$H_{11}^{fr}+H_{12}^{fr}+H_{23}^{fr}+H_{24}^{fr}$	7
4	w/	w/o	w/o	w/o	w/o	$A(H_{11}^{fs}+H_{12}^{fs}+H_{23}^{fs}+H_{24}^{fs})A^T$	8
5	w/	w/	w/o	w/o	w/	$A(H_{11}^{fr}+H_{12}^{fs}+H_{23}^{fs}+H_{24}^{fr})A^T$	9
6	w/	w/	w/	w/	w/	$A(H_{11}^{fr}+H_{12}^{fr}+H_{23}^{fr}+H_{24}^{fr})A^T$	10

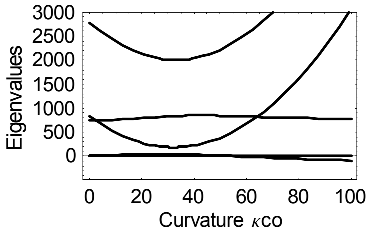


(a)



(b)

Figure 5: Case 1



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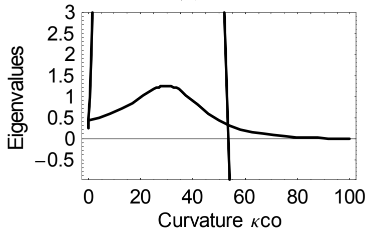
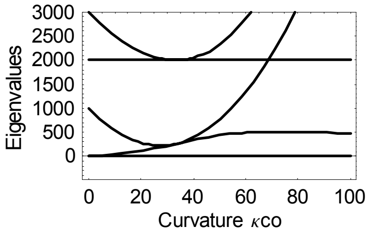
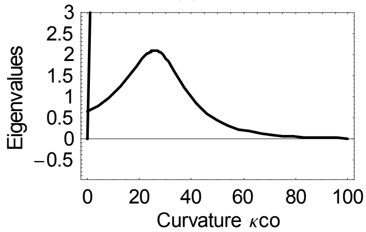


(b)

Figure 6: Case 2

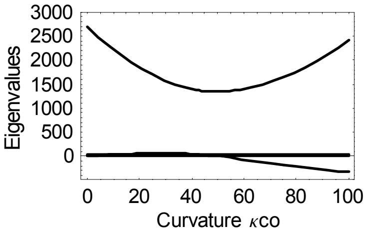


(a)

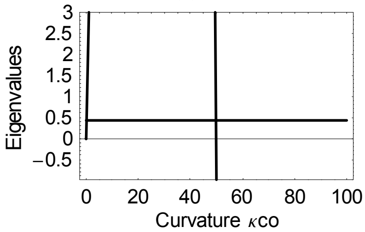


(b)

Figure 7: Case 3

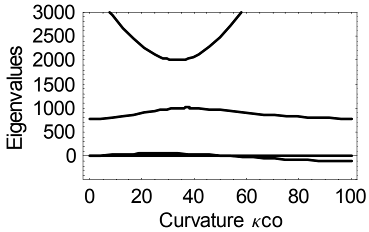


(a)



(b)

Figure 8: Case 4



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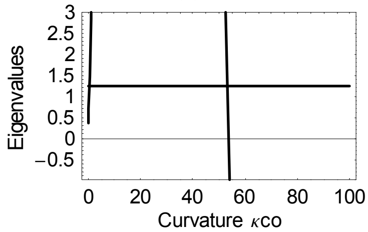
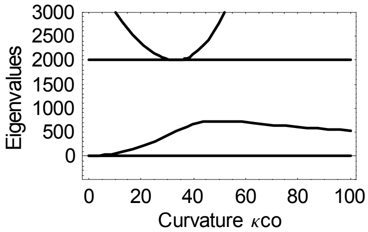


(b)

Figure 9: Case 5



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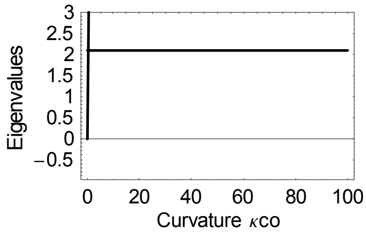

(b)

Figure 10: Case 6