Identification of Contact Conditions from Position and Velocity Information

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Abstract

This paper proposes an algorithm for identification of contact conditions between a grasped object and external environment from position and velocity information of a robot hand. The contact conditions mean contact type, contact position, and contact force. Soft finger contact type, line contact type, and plane contact type are considered in addition to point contact type. The contact type is identified by rank of the matrix that places estimated angular velocity of the object at contact point. Contact position, contact normal and contact line are also estimated. This method has the following merits as compared with the method using 6-axes force information described in the previous paper. The identification problem is reduced to the problem of solving linear equations. The required number of active sensing motion is less.

Key Words: Identification, Contact Conditions, Active Sensing, Position and Velocity Information, Hand, Grasped object and external environment

1. Introduction

When a grasped object is in contact with external environment as shown in Fig. 1, it is required to identify contact conditions prior to performing assembly tasks. Human can easily identify current contact conditions by using information from one's sensitive hand and accomplish the tasks. By changing contact type, one can make a fine positioning and assembling task. Therefore, it is necessary for realization of human action on the robot to identify and control the contact conditions that mean contact types, contact position, and contact forces.

There are various researches about contact conditions using force and/or velocity information. Hyde et al. [4] and Akella et al. [1] proposed a method for controlling contact transition from free motion to constrained motion. Dutre et al. [3] identified geometrical uncertainties and detected topological transitions in contact situations. Newman et al. [15] investigated a method of interpretation of force and moment data for peg-in-hole assembly. Tsujimura et al. [18], Kaneko et al. [5] and Ueno et al. [19] detected contact position by probe, active antenna and multiple active antennas, respectively. Bicchi et al. [2] identified contact position and contact normal direction in case of point and soft finger contact types. Also, Zhou et al. [20] localized contact in case of point contact type and inferred accuracy of the proposed approach. Salisbury et al. [17] proposed a method for identification of contact position on the sensor surface using multiple active sensing. Nagata et al. [14] and Kitagaki et al. [7] proposed methods for identifying the contact position of the unknown object by using active sensing. In these methods, however, it was assumed that the contact condition is of point contact type with friction. Oussalah [16] proposed a method for identifying the contact normal by force and velocity without friction. Muto and Shimokura [13] detected contact position and contact normal by using force and velocity information complementarily. Kikuuwe and Yoshikawa [6] described an algorithm for robot perception of impedance. This method

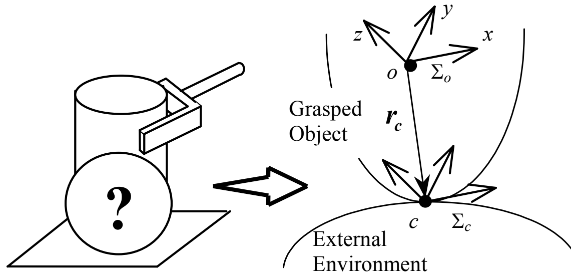

Fig. 1: Interaction between a grasped object and external environment

estimates mechanical impedance of external environment during arbitrary manipulation. Unknown parameters such as elasticity, viscosity, inertia matrices, and contact position are roughly estimated. However, these works did not identify contact type such as soft finger, line, and plane contact type.

The authors [9]-[11] discussed identification of contact conditions including soft finger contact type, line contact type and plane contact type in addition to the point contact type. We proposed an algorithm for identification of contact conditions from contaminated data of contact force and moment. In this algorithm, contact position and contact moment are estimated by the least squares method, and then contact type is identified by eigenvalues of covariance matrix of estimated contact moment.

Human hand can measure not only 6-axes force information but also position and velocity information. However, Ref. [9]-[11] did not utilize this information effectively. For this reason, this paper treats an identification problem of contact conditions from position and velocity information. We direct our attention to the difference of angular velocity at contact point between a grasped object and external environment. This constraint is dual to the constraint of moment generated at contact point. From this constraint, four contact types can be classified and contact position can be estimated. Moreover, the directions of contact normal and contact line are also estimated.

2. Problem Formulation

2.1. Notations

Suppose that a grasped object is in contact with external environment, as shown in Fig. 1. We define the following symbols shown in Fig. 2.

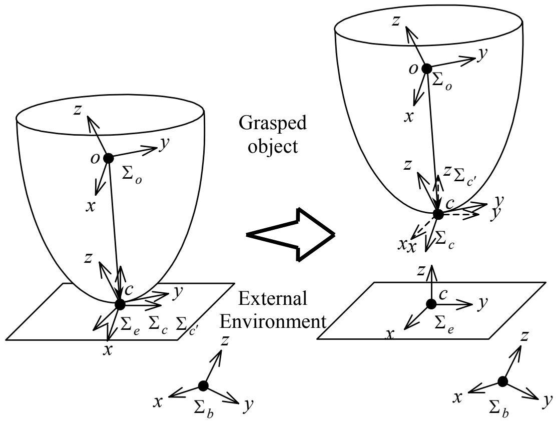

Fig.2: Coordinate frames

c: contact point between the grasped object and external environment

o: origin of a sensor mounted on a robot hand.

Σ_b : base coordinate frame.

Σ_e: contact coordinate frame fixed at c on the external environment.

Σ_o : object (sensor, or hand) coordinate frame fixed at o

Σ_c : contact coordinate frame fixed at c on the object. The orientation of this frame is the same as that of Σ_o .

$\Sigma_{c'}$: contact coordinate frame fixed at c on the object. The initial orientation of this frame is the same as that of Σ_e . This frame is classified by contact type.

${}^o\boldsymbol{r}_c$: contact position with respect to Σ_o .

The frames Σ_o, Σ_c, and $\Sigma_{c'}$ are already defined in Ref. [11]. The frames Σ_b and Σ_e are newly defined in this paper, because relative velocity between the object and external environment will be derived in order to classify contact types. The position of Σ_c is unknown, then ${}^o\boldsymbol{r}_c$ is also unknown. The configuration of $\Sigma_{c'}$ and Σ_e are unknown. Note that the orientation of $\Sigma_{c'}$ is different from that of Σ_c . In this paper, the term of configuration means position and orientation.

2.2. Homogeneous Transform Matrix

The configuration of Σ_o with respect to Σ_b is denoted by a homogeneous transform matrix ${}^bT_o$:

$$ {}^bT_o(t_i) = \begin{bmatrix} {}^bR_o(t_i) & {}^b\boldsymbol{p}_{b,o}(t_i) \\ 0 & 1 \end{bmatrix}, \tag{1} $$

where ${}^b\boldsymbol{p}_{b,o}$ is a position vector from the origin of Σ_b to the origin of Σ_o with respect to Σ_b. The letter ${}^bR_o$ is a rotation matrix of Σ_o with respect to Σ_b. The letter t_i means time at the i-th sample. In a similar way, the following matrices are utilized.

$$ {}^bT_e = \begin{bmatrix} {}^bR_e & {}^b\boldsymbol{p}_{b,e} \\ 0 & 1 \end{bmatrix}, \quad {}^eT_{c'}(t_i) = \begin{bmatrix} {}^eR_{c'}(t_i) & 0 \\ 0 & 1 \end{bmatrix}, $$

$$ {}^{c'}T_c = \begin{bmatrix} {}^{c'}R_c & 0 \\ 0 & 1 \end{bmatrix}, \quad {}^cT_o = \begin{bmatrix} I_3 & -{}^o\boldsymbol{r}_c \\ 0 & 1 \end{bmatrix} \tag{2} $$

Since ${}^bT_o(t_i)$ and ${}^eT_{c'}(t_i)$ change due to the active sensing motion, these depend on time. The vectors ${}^e\boldsymbol{p}_{e,c'}(t_i)$ and ${}^{c'}\boldsymbol{p}_{c',c}$ become zero, because position of the origin of frames Σ_e, $\Sigma_{c'}$, and Σ_c are the same as each other. The rotation matrix ${}^cR_o$ becomes the 3×3 identity matrix I_3, because the orientation of Σ_c and Σ_o are the same. The position vector ${}^c\boldsymbol{p}_{c,o}$ is expressed by ${}^c\boldsymbol{p}_{c,o} = -{}^o\boldsymbol{r}_c$.

2.3. Assumptions

We make the following assumptions for clarification of discussions.

(A1) An object is firmly grasped and manipulated by a robot hand.

(A2) The configuration ${}^{b}T_{o}(t_i)$ and the time derivative ${}^{b}\dot{T}_{o}(t_i)$ are known and noiseless.

(A3) The grasped object is in contact with external environment through point, soft finger, line, or plane (face) contact type with friction. The contact position and type are unknown.

(A4) The contact type remains unchanged during identification process.

(A5) No slip occurs at the contact position c.

Assumption (A2) means that, for an example, ${}^{b}T_{o}(t_i)$ and ${}^{b}\dot{T}_{o}(t_i)$ can be measured from angle and velocity of joint sensors mounted on the robot arm. Assumption (A3) is illustrated in Fig. 3. Contact types are classified by DOF of contact [8]. DOF is denoted by m.

$m = 0$: Plane contact type with friction

$m = 1$: Line contact type with friction

$m = 2$: Soft finger contact type with friction.
This contact type doesn't exist between rigid objects.

$m = 3$: Point contact type with friction.

Assumption (A5) implies that contact force is generated inside the friction cone.

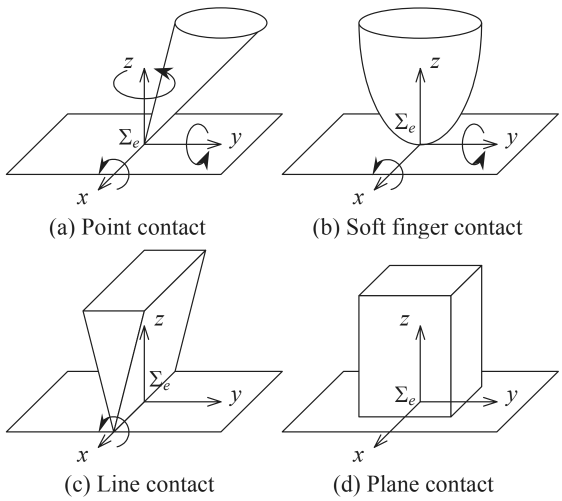

Fig. 3: Contact types

2.4. Constraint of angular velocity

We consider 4 contact types described in Assumption (A3). These contact types can be characterized by using a coordinate frame Σ_e whose z axis is aligned with the normal at the contact point on the external environment. Added to this, in case of line contact type, x axis of Σ_e is aligned with contact line. The angular velocity of the object at the contact point with respect to the frame Σ_e is denoted by $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ and is constrained by

$$\left.\begin{array}{ll} \text{Point contact} & : \text{no constraint} \\ \text{Soft finger contact} & : \omega_z = 0 \\ \text{Line contact} & : \omega_y = \omega_z = 0 \\ \text{Plane contact} & : \omega_x = \omega_y = \omega_z = 0 \end{array}\right\} \quad (3)$$

Note that the constraint of $\boldsymbol{\omega}$ is dual to that of contact moment. This means that if moment around a certain axis can be applied then angular velocity around this axis cannot be generated and if moment cannot be applied, angular velocity can be generated

3. Identification of contact conditions

3.1. Fundamental equations

The configuration of Σ_o with respect to Σ_b is obtained by

$${}^{b}T_{o}(t_i) = \{{}^{b}T_{e}\}\{{}^{e}T_{c'}(t_i)\}\{{}^{c'}T_{c}\}\{{}^{c}T_{o}\}. \quad (4)$$

The time derivative of Eq. (4) is provided by

$${}^{b}\dot{T}_{o}(t_i) = \{{}^{b}T_{e}\}\{{}^{e}\hat{V}_{e,c'}(t_i)\}\{{}^{e}T_{c'}(t_i)\}\{{}^{c'}T_{c}\}\{{}^{c}T_{o}\}, \quad (5)$$

where ${}^{e}\hat{V}_{e,c'}(t_i)$ is the spatial velocity of $\Sigma_{c'}$ with respect to Σ_e [12], which is represented by

$${}^{e}\hat{V}_{e,c'}(t_i) := \{{}^{e}\dot{T}_{c'}(t_i)\}\{{}^{e}T_{c'}(t_i)\}^{-1} = \begin{bmatrix} {}^{e}\Omega_{e,c'}(t_i) & 0 \\ 0 & 0 \end{bmatrix}. \quad (6)$$

The matrix ${}^{e}\Omega_{e,c'}$ is a skew symmetric matrix:

$${}^{e}\Omega_{e,c'} := \begin{bmatrix} 0 & -{}^{e}\omega_{e,c'y}(t_i) & {}^{e}\omega_{e,c'z}(t_i) \\ {}^{e}\omega_{e,c'y}(t_i) & 0 & -{}^{e}\omega_{e,c'x}(t_i) \\ -{}^{e}\omega_{e,c'z}(t_i) & {}^{e}\omega_{e,c'x}(t_i) & 0 \end{bmatrix}. \quad (7)$$

From Eq. (5), we have the following two equations:

$${}^{b}\dot{R}_{o}(t_i) = \{{}^{b}R_{e}\}\{{}^{e}\Omega_{e,c'}(t_i)\}\{{}^{e}R_{c'}(t_i)\}\{{}^{c'}R_{c}\} \quad (8)$$

$${}^{b}\dot{\boldsymbol{p}}_{b,o}(t_i) = -\{{}^{b}\dot{R}_{o}(t_i)\}\{{}^{o}\boldsymbol{r}_{c}\} \quad (9)$$

We will identify contact type and contact position by Eqs. (8) and (9), respectively.

3.2. Contact type

Contact types are classified from Eq. (8). A skew symmetric matrix of relative velocity from Σ_e to $\Sigma_{c'}$ with respect to Σ_b is given by

$$ {}^{b}\Omega_{e,c'}(t_i) = \{{}^{b}R_e\}\{{}^{e}\Omega_{e,c'}(t_i)\}\{{}^{b}R_e\}^{-1} \tag{10} $$

Since the orientation of Σ_o is the same as that of Σ_c, we have

$$ {}^{b}R_c(t_i) = {}^{b}R_o(t_i) . \tag{11} $$

From Eqs. (8), (10), and (11), we obtain

$$ {}^{b}\Omega_{e,c'}(t_i) = \{{}^{b}\dot{R}_o(t_i)\}\{{}^{b}R_o(t_i)\}^{-1} . \tag{12} $$

Transforming Eq. (12) to a three dimensional vector yields

$$ {}^{b}\boldsymbol{\omega}_{e,c'}(t_i) = [\{{}^{b}\dot{R}_o(t_i)\}\{{}^{b}R_o(t_i)\}^{-1}]^{\vee} , \tag{13} $$

where $\bullet^{\vee}$ represents a transform from a skew symmetric matrix $\bullet$ to a 3 dimensional vector. We define the matrix M that places ${}^{b}\boldsymbol{\omega}_{e,c'}(t_i)$ as the following form:

$$ M := [{}^{b}\boldsymbol{\omega}_{e,c'}(t_1) \quad {}^{b}\boldsymbol{\omega}_{e,c'}(t_2) \cdots {}^{b}\boldsymbol{\omega}_{e,c'}(t_k)] \in \Re^{3\times k} , \tag{14} $$

Since ${}^{b}\boldsymbol{\omega}_{e,c'}(t_i)$ is written as

$$ {}^{b}\boldsymbol{\omega}_{e,c'}(t_i) = \{{}^{b}R_e\}\{{}^{e}\boldsymbol{\omega}_{e,c'}(t_i)\} \tag{15} $$

and the matrix ${}^{b}R_e$ is unknown but nonsingular, we have

$$ \mathrm{rank}M = \mathrm{rank}[{}^{e}\boldsymbol{\omega}_{e,c'}(t_1) \quad {}^{e}\boldsymbol{\omega}_{e,c'}(t_2) \cdots {}^{e}\boldsymbol{\omega}_{e,c'}(t_k)] . \tag{16} $$

${}^{e}\boldsymbol{\omega}_{e,c'}(t_i)$ is constrained by Eq. (3), then rank of M becomes

$$ \mathrm{rank}\, M = \begin{cases} 3 & \text{for Point contact} \\ 2 & \text{for Soft finger contact} \\ 1 & \text{for Line contact} \\ 0 & \text{for Plane contact} \end{cases} \tag{17} $$

Therefore, contact types are classified by analyzing rank of M. From the above discussion, 3 times of active sensing motion are required in order to judge which contact type happens.

3.3. Estimation of direction of contact normal and contact line

The direction of contact normal or contact line must be estimated if the contact type is judged as soft finger contact type or line contact type by the method described in Section. 3.2

3.3.1. Direction of contact normal

In case of soft finger contact type, Eq. (15) becomes

$$ {}^{b}\boldsymbol{\omega}_{e,c'}(t_i) = ({}^{b}\boldsymbol{r}_{ex})\{{}^{e}\omega_{e,c'x}(t_i)\} + ({}^{b}\boldsymbol{r}_{ey})\{{}^{e}\omega_{e,c'y}(t_i)\} , \tag{18} $$

where

$$ {}^{b}R_e = [{}^{b}\boldsymbol{r}_{ex} \quad {}^{b}\boldsymbol{r}_{ey} \quad {}^{b}\boldsymbol{r}_{ez}] . $$

The direction of the normal vector is aligned with the direction which cannot generate angular velocity, that is, the direction is tangent to both ${}^{b}\boldsymbol{r}_{ex}$ and ${}^{b}\boldsymbol{r}_{ey}$. If we use independent two vectors ${}^{b}\boldsymbol{\omega}_{e,c'}(t_i)$ and ${}^{b}\boldsymbol{\omega}_{e,c'}(t_j)$, the direction of contact normal $\boldsymbol{n}$ is calculated by

$$ \boldsymbol{n} = \frac{\{{}^{b}\boldsymbol{\omega}_{e,c'}(t_i)\}\times\{{}^{b}\boldsymbol{\omega}_{e,c'}(t_j)\}}{\left\|\{{}^{b}\boldsymbol{\omega}_{e,c'}(t_i)\}\times\{{}^{b}\boldsymbol{\omega}_{e,c'}(t_j)\}\right\|} \tag{19} $$

3.3.2. Direction of contact line

In case of line contact type, Eq. (15) becomes

$$ {}^{b}\boldsymbol{\omega}_{e,c'}(t_i) = ({}^{b}\boldsymbol{r}_{ex})\{{}^{e}\omega_{e,c'x}(t_i)\} . \tag{20} $$

In this case, the direction of angular velocity is that of contact line $\boldsymbol{l}$ which is obtained by

$$ \boldsymbol{l} = {}^{b}\boldsymbol{\omega}_{e,c'}(t_i) \Big/ \left\|{}^{b}\boldsymbol{\omega}_{e,c'}(t_i)\right\| . \tag{21} $$

3.4. Estimation of contact position

The contact position will be estimated by using Eq. (9). From Assumption (A2), although ${}^{b}\dot{\boldsymbol{p}}_{b,o}(t_i)$ and ${}^{b}\dot{R}_o(t_i)$ are known, ${}^{o}\boldsymbol{r}_c$ is unknown. By using

$$ A_i := {}^{b}\dot{R}_o(t_i) , \quad \boldsymbol{x} := {}^{o}\boldsymbol{r}_c , \quad \boldsymbol{b}_i := -{}^{b}\dot{\boldsymbol{p}}_{b,o}(t_i) , $$

Eq. (9) becomes

$$ A_i \boldsymbol{x} = \boldsymbol{b}_i . \tag{22} $$

Eq. (22) has three unknown parameters and three equations. However, the matrix A_i has a skew symmetric matrix ${}^{e}\Omega_{e,c'}(t_i)$, then A_i is singular.

$$ \mathrm{rank}\, A_{\mathrm{i}} = \mathrm{rank}\,{}^{e}\Omega_{e,c'}(t_i) \le 2 . \tag{23} $$

The contact position cannot be estimated by 1 time of active sensing motion. From several times of active sensing motion, we obtain

$$ A\boldsymbol{x} = \boldsymbol{b} \tag{24} $$

where

$$ A := \begin{bmatrix} A_1 \\ \vdots \\ A_k \end{bmatrix} \in \Re^{3k\times 3} , \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \vdots \\ \boldsymbol{b}_k \end{bmatrix} \in \Re^{3k} $$

3.4.1. Case of Point or Soft finger contact type

In case of point contact type, any angular velocity occurs at the contact point. In case of soft finger contact type, angular velocity around x and y axes can be generated. Hence, we have

$$ \mathrm{rank}\, A = 3 \quad \text{for} \quad i \ge 2 . \tag{25} $$

Contact position can be estimated uniquely.

3.4.2. Case of Line contact type

In case of line contact type, motion is constrained around x axis of the frame Σ_e, then

$$\operatorname{rank} A = 2 \tag{26}$$

(refer to Appendix). Eliminating Eq. (24) yields the following form:

$$\begin{bmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ 0 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}, \tag{27}$$

where $\boldsymbol{a}_1, \boldsymbol{a}_2 \in \Re^3$ and $b_1, b_2 \in \Re$ are appropriate values. Hence, the contact position can be estimated as

$$\hat{\boldsymbol{x}} = [\boldsymbol{a}_1 \ \ \boldsymbol{a}_2] \begin{bmatrix} \boldsymbol{a}_1^T \boldsymbol{a}_1 & \boldsymbol{a}_1^T \boldsymbol{a}_2 \\ \boldsymbol{a}_2^T \boldsymbol{a}_1 & \boldsymbol{a}_2^T \boldsymbol{a}_2 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \alpha(\boldsymbol{a}_1 \times \boldsymbol{a}_2), \tag{28}$$

where α takes any value. In case of line contact type, there exits a free parameter along contact line.

3.4.3. Plane (face) contact type

In case of plane contact type,

$$\operatorname{rank} A = 0 \tag{29}$$

because of ${}^e\Omega_{e,c'}(t_i) = 0$. Hence, contact position cannot be estimated.

3.5. Algorithm

Based on Section 3.3 and 3.4, we propose an algorithm for identification of contact conditions from position and velocity information.

Step 1: Command 3 times of active sensing motion and measure the configuration ${}^b\boldsymbol{p}_{b,o}(t_i)$, ${}^bR_o(t_i)$ and the time derivative ${}^b\dot{\boldsymbol{p}}_{b,o}(t_i)$, ${}^b\dot{R}_o(t_i)$.

Step 2: Calculate ${}^b\boldsymbol{\omega}_{e,c'}(t_i)$ by Eq. (13).

Step 3: Generate *M* by Eq. (14).

Step 4: Judge which contact type happens by using the rank of *M*.

Step 5: If it is judged as Point contact type, then calculate ${}^o\boldsymbol{r}_c$ by Section 3.4.1.

Step 6: If it is judged as Soft finger contact type, then calculate the direction of normal by Eq. (19) and ${}^o\boldsymbol{r}_c$ by Section 3.4.1.

Step 7: If it is judged as Line contact type, then calculate the direction of contact line by Eq. (21) and the contact position ${}^o\boldsymbol{r}_c$ by Section 3.4.2.

In case of plane contact type, unknown parameters cannot be estimated, but contact type can be judged. This algorithm can estimate not only contact type and contact position but also the direction of contact normal and contact line.

4. Conclusions

This paper has proposed an algorithm for identification of contact conditions from position and velocity information. The contributions of this paper are as follows:

(1) It is shown that angular velocity at contact point with respect to the base frame can be obtained by measuring position and velocity information of the robot hand. From rank of the matrix that places the angular velocity, the four contact types can be judged. We make it clear that three times of active sensing motion are required in order to the classification.

(2) The direction of contact normal or contact line has been derived for soft finger contact type or line contact type, respectively.

(3) It is shown that contact position is estimated uniquely for point and soft finger contact type, but it is determined with one degree of freedom for line contact type.

It has been analytically shown that the contact conditions can be obtained from position and velocity information. Since our proposed method uses position and velocity information only, it cannot estimate contact force and moment but has the following merits: (1) The identification of contact conditions is reduced to the problem of solving linear equations. (2) The required number of active sensing motion is less than that of the method from force and moment information.

In the future work, we treat the case where the data are contaminated with noise. We will investigate sensor fusion using velocity information described in this paper and force information treated in Ref. [11] in order to obtain more accurate and rapid estimates.

Appendix

In case of line contact type, the matrix ${}^eR_{c'}(t_i)$ is represented as

$${}^eR_{c'}(t_i) = \mathrm{R}_x(\theta_i), \tag{30}$$

where $\mathrm{R}_x(\bullet)$ is a rotation matrix around x axis:

$$\mathrm{R}_x(\bullet) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\bullet) & -\sin(\bullet) \\ 0 & \sin(\bullet) & \cos(\bullet) \end{bmatrix}$$

and θ_i implies relative angle from Σ_e to $\Sigma_{c'}$ at time

t_i. Hence, Eq. (8) becomes

$$A_i = {}^e\omega_{e,c'x}(t_i)({}^bR_e)\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta_i & -\cos\theta_i \\ 0 & \cos\theta_i & -\sin\theta_i \end{bmatrix}({}^{c'}R_c) \quad (31)$$

From Eq. (21), the direction of contact line, ${}^b\boldsymbol{r}_{ex}$, is known, then the estimated value of ${}^bR_e$ can be written by

$$\begin{aligned} {}^b\hat{R}_e &= [{}^b\boldsymbol{r}_{ex},\ {}^b\boldsymbol{r}_{ey}\cos\phi + {}^b\boldsymbol{r}_{ez}\sin\phi,\ -{}^b\boldsymbol{r}_{ey}\sin\phi + {}^b\boldsymbol{r}_{ez}\cos\phi] \\ &= {}^bR_e\,\mathrm{R}_x(\phi) \end{aligned} \quad (32)$$

where ϕ is an arbitrary value. From Eqs. (31) and (32), we have

$$\begin{aligned} &({}^b\hat{R}_e)^T(A_i) \\ &= {}^e\omega_{e,c'x}(t_i)\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin(\theta_i-\phi) & -\cos(\theta_i-\phi) \\ 0 & \cos(\theta_i-\phi) & -\sin(\theta_i-\phi) \end{bmatrix}({}^{c'}R_c) = \begin{bmatrix} 0 \\ \overline{A}_i \end{bmatrix} \end{aligned} \quad (33)$$

and

$$({}^b\hat{R}_e)^T(b_i) = -({}^b\hat{R}_e)^T(A_i)({}^c\boldsymbol{r}_o^*) = \begin{bmatrix} 0 \\ \overline{\boldsymbol{b}}_i \end{bmatrix} \quad (34)$$

where the matrix $\overline{A}_i \in \Re^{2\times 3}$ and the vector $\overline{\boldsymbol{b}}_i \in \Re^2$ are appropriate values. In Eq. (33), the data at time t_i is dependent of that of t_j, because

$$({}^b\hat{R}_e)^T(A_i) = \Theta_{i,j}({}^b\hat{R}_e)^T(A_j) \quad (35)$$

where

$$\Theta_{i,j} = \frac{{}^e\omega_{e,c'x}(t_i)}{{}^e\omega_{e,c'x}(t_j)}\mathrm{R}_x(\theta_i-\theta_j). \quad (36)$$

Therefore, Eq. (24) can be eliminated to the form of Eq. (27).

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