# Stability Analysis of 3D Grasps by A Multifingered Hand

Takayoshi YAMADA*, Tarou KOISHIKURA*, Yuto MIZUNO*,
Nobuharu MIMURA**, Yasuyuki FUNAHASHI*
*: Nagoya Institute of Technology, Gokiso-cho, Showa-Ku, Nagoya 466-8555, Japan
**: Niigata University, Ninomachi, Ikarashi, Niigata 950-2181, Japan
yamat@eine.mech.nitech.ac.jp

## Abstract

*In this paper, we discuss stability of 3D grasps by using a three-dimensional spring model (3D spring model). Many works described stability of frictionless grasp. These works used a one-dimensional spring model (1D spring model) for representation of frictionless contact as a matter of course. However, 1D spring model involves the following two problems.* (i) *Displacement of finger is restricted to one dimension along the initial normal at contact point.* (ii) *It is not clearly considered that a finger generates contact force to the object along the normal direction only. To overcome the problems and provide accurate grasp stability, we introduce 3D spring model. Then finger's displacement is relaxed and finger's contact force is precisely formulated. We analyze grasp stability from the viewpoint of the potential energy method. From numerical examples, we show that there exists an optimum contact force for stable grasp. Moreover, we also analyze frictional grasp by using contact kinematics of pure rolling. By comparing frictional grasp stability with frictionless one, we prove that friction enhances stability of grasp.*

## 1 Introduction

A multifingered robot hand has potential ability of fine and dexterous manipulation. Stability is the tendency of a system to return to an equilibrium state when displaced from this state. It is required for the hand to maintain the grasp stable against external disturbances. This paper discusses stability of three-dimensional grasp.

Many works [1]-[7][9]-[12][14]-[21] explored stability of grasps. Grasp stability depends on whether friction exists or not, because finger's motion is different as shown in Fig. 1.

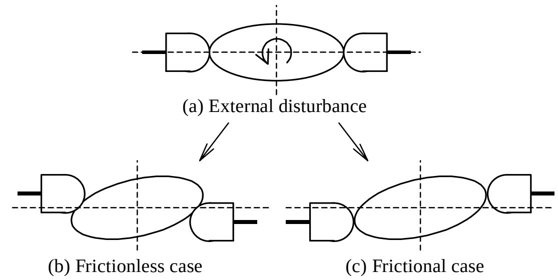



Figure 1: Difference of finger's displacement between frictionless contact and frictional contact for 2D grasp

### 1.1 Frictional Grasps

The following papers discussed frictional grasp stability.

Nguyen [14], Kaneko et al. [7] analyzed grasp stability by replacing a finger with a virtual spring model. The stability is evaluated by positive definiteness of a hessian of potential energy stored in the grasp system. The hessian is called a grasp stiffness matrix.

Howard and Kumar [5], Funahashi et al. [4], Yamada et al. [20], and Choi et al. [2] included curvature effect of contact point. Sugar and Kumar [18] explored metrics of robustness of errors. Svinin [19] derived range of contact force for rotational stability.

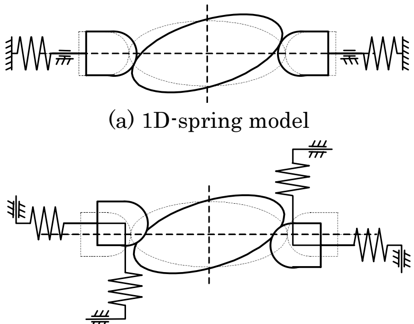
Cutkosky and Kao [3] included the compliace of the finger joints and provided a systematic method for deriving grasp stability. Howard and Kumar [6] explored an enveloping grasp.

Montana [12], Shimoga [17], Xiong [21] discussed dyamamic stability of the grasp.

### 1.2 Frictionless Grasps

When the hand grasps a cake of soap or a cube of ice, a coefficient of contact friction is small. Hence, it is important to analyze frictionless grasp stability.

Refs. [4] [5] [6] [14] also explored frcitionless grasp stability. Rimon and Burdick [15], Lin et al. [9] [10] presented systematic approach. These papers used 1D spring model for representation of frictionless contact as a matter of course (Fig. 2). However, these papers did not consider that contact force act along the normal



(a) 1D-spring model

(b) 2D-spring model

Figure 2: Difference of finger's displacement betweem 1D spring model and 2D spring model for frictionless 2D grasp

direction, and tangential component of the contact force is equal to zero when the grasped object is displaced. Ref. [2] did not considered the effect of initial grasping force.

To overcome the problems, Saha et al. [16] introduced 2D spring model as shown in Fig.2(b) and explicitly formulated the frictionless contact force. Then, more accurate evaluation of grasp stability was obtained.

### 1.3 Approach of this paper

This paper discusses stability of 3D grasps, based on Ref. [16]. 3D spring model is introduced into frictionless 3D grasps. The frictionless contact force in 3D is explicitly formulated. And grasp stability is determined by the potential energy method. From numerical examples, it is shown that there exists a region of contact force for stable grasp and an optimum contact force. Moreover, frictional grasp stability is also investigated by using contact kinematics of pure rolling. By comparing frictional grasp stability with frictionless one, we prove that friction enhances stability of grasp.

## 2 Formulation

Suppose that an object is grasped by a multifingered hand as shown in Fig.3. We explore stability of the grasp.

### 2.1 Assumptions

In this paper, the following assumptions are considered.

(A1) The geometries of the fingertips and the object are known and rigid. Finger is shaped a sphere with curvature $\kappa_{fi}$. Local geometry of the object at the contact point is shaped a second-order model with primary curvatures $\kappa_{ai}$ and $\kappa_{bi}$ whose principal axes

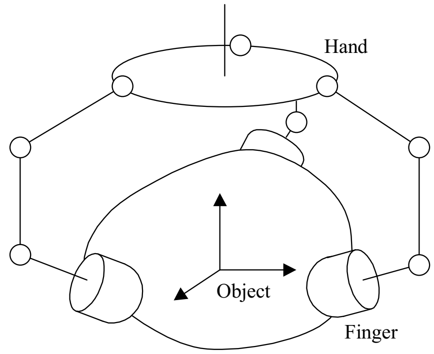



Figure 3: 3D grasp system

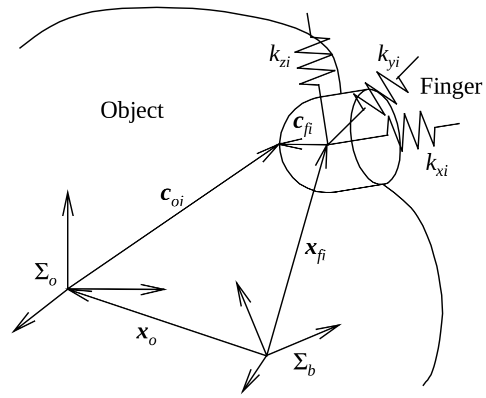



Figure 4: Coordinate frame

are denoted by $\boldsymbol{r}_{zi}$ and $\boldsymbol{r}_{yi}$, respectively. (A2) Initial configuration is known and is in equilibrium. (A3) Infinitesimal displacement is occurred in the object due to external disturbances. The displacement is denoted by $\boldsymbol{\epsilon} = [x, y, z, \xi, \eta, \zeta]^T$. (A4) Finger's displacement is infinitesimal, and the relationship between finger's displacement and reaction force is replaced with 3D linear spring model which is fixed at center of the fingertip as shown in Fig.4. Stiffness of the spring is programmable. One spring $k_{xi}(\geq 0)$ is set along the normal and the others $k_{yi}(\geq 0)$ and $k_{zi}(\geq 0)$ are parallel to the tangent. $k_{yi}$ and $k_{zi}$ are parallel to the primary curvatures $\kappa_{bi}$ and $\kappa_{ai}$, respectively.

### 2.2 Notations

Reference coordinate and the i-th finger's coordinate are denoted by $\Sigma_b$ and $\Sigma_{fi}$, respectively. $\Sigma_{fi}$ is aligned with the 3D spring model. $x$ axis of $\Sigma_{fi}$ is aligned with the inward normal of the finger. Position and orientation of $\Sigma_{fi}$ with respect to $\Sigma_b$ are denoted by $\boldsymbol{x}_{fi}$ and $R_{fi} := [\boldsymbol{r}_{xi}, \boldsymbol{r}_{yi}, \boldsymbol{r}_{zi}]$. Contact position on the fingertip is given by

$$\boldsymbol{c}_{fi}(\boldsymbol{u}_{fi}) := \kappa_{fi}^{-1}\begin{bmatrix}\cos u_{fi}\cos v_{fi}\\ \cos u_{fi}\sin v_{fi}\\ \sin u_{fi}\end{bmatrix}, \qquad (1)$$

where $\boldsymbol{u}_{fi} = [u_{fi}, v_{fi}]^T$ is parameter of contact position on the $i$-th finger. The inward unit normal at $\boldsymbol{c}_{fi}$ is

$$\boldsymbol{n}_{fi}(\boldsymbol{u}_{fi}) := -\kappa_{fi}\boldsymbol{c}_{fi}(\boldsymbol{u}_{fi}). \qquad (2)$$

Object coordinate is denoted by $\Sigma_o$. $\Sigma_b$ is aligned with the initial configuration of $\Sigma_o$. When the object is displaced by disturbances, position and orientation of $\Sigma_o$ is given by

$$\boldsymbol{x}_o = [x, y, z]^T,\ R_o = \mathrm{Rot}(\xi, \eta, \zeta), \qquad (3)$$

where $\mathrm{Rot}(\bullet)$ is a 3D rotation matrix. Object local coordinate at contact point is represented by $\Sigma_{oi}$. Contact position of the object is $\boldsymbol{c}_{oi}$. Position and Orientation of $\Sigma_{oi}$ with respect to $\Sigma_o$ is denoted by $\boldsymbol{c}_i$ and $R_{ci}$. Local shape around the contact point is approximated by the primary curvatures $\kappa_{ai}$ and $\kappa_{bi}$.

$$\begin{aligned} \boldsymbol{c}_{oi}(\boldsymbol{u}_{ci}) &:= \boldsymbol{c}_i + R_{ci}\boldsymbol{d}_i(\boldsymbol{u}_{ci}), && (4)\\ \boldsymbol{d}_i(\boldsymbol{u}_{ci}) &:= [u_{ci},\ v_{ci},\ (\kappa_{ai}u_{ci}^2 + \kappa_{bi}v_{ci}^2)/2]^T, && (5)\\ R_{ci} &:= [\boldsymbol{r}_{zi}, \boldsymbol{r}_{yi}, -\boldsymbol{r}_{xi}], && (6)\end{aligned}$$

where $\boldsymbol{u}_{ci} = [u_{ci}, v_{ci}]^T$ is parameter of contact position on the grasped object. The inward unit normal vector is denoted by $\boldsymbol{n}_{oi}$.

$$\begin{aligned} \boldsymbol{n}_{oi}(\boldsymbol{u}_{ci}) &= (\boldsymbol{d}_{iu} \otimes \boldsymbol{d}_{iv})/\|\boldsymbol{d}_{iu} \otimes \boldsymbol{d}_{iv}\| \\ &= \frac{1}{\sqrt{\kappa_{ai}^2 u_{ci}^2 + \kappa_{bi}^2 v_{ci}^2 + 1}}\begin{bmatrix}-\kappa_{ai}u_{ci}\\ -\kappa_{bi}v_{ci}\\ 1\end{bmatrix}, \qquad (7)\end{aligned}$$

where $\boldsymbol{d}_{iu} := \partial\boldsymbol{d}_i/\partial u_{ci}$, $\boldsymbol{d}_{iv}/ = \partial\boldsymbol{d}_i/\partial v_{ci}$, and $\otimes$ is cross product.

## 2.3 Spring Compression

From kinematics of contact, we have the following two equations. One is a condition of contact point.

$$\boldsymbol{x}_o + R_o\boldsymbol{c}_{oi} = \boldsymbol{x}_{fi} + R_{fi}\boldsymbol{c}_{fi} \qquad (8)$$

Another is a condition of the normal direction.

$$R_o\boldsymbol{n}_{oi} = -R_{fi}\boldsymbol{n}_{fi} \qquad (9)$$

Due to object displacement $\boldsymbol{\epsilon}$, compression of the virtual spring is given by

$$\begin{aligned} \boldsymbol{\delta}_i :=& R_{fi}^T\{\boldsymbol{x}_{fi}(\boldsymbol{\epsilon}) - \boldsymbol{x}_{fi}(0)\} \\ =& R_{fi}^T\{\boldsymbol{x}_o + R_o\boldsymbol{c}_{oi} + \kappa_{fi}^{-1}R_o\boldsymbol{n}_{oi} - \boldsymbol{x}_{fi}(0)\}, \qquad (10)\end{aligned}$$

where note that $\boldsymbol{\delta}_i = \boldsymbol{\delta}_i(\boldsymbol{\epsilon}, \boldsymbol{u}_{ci})$ because $\boldsymbol{c}_{oi}$ and $\boldsymbol{n}_{oi}$ are functions of $\boldsymbol{u}_{ci}$. The relationship between $\boldsymbol{\epsilon}$ and $\boldsymbol{u}_{ci}$ depends on whether friction exists or not. This relationship is described in Section 3.1 and 3.2.

## 2.4 Potential Energy

The initial contact force with respect to $\Sigma_{fi}$ is expressed as $\boldsymbol{f}_i := [f_{xi}, f_{yi}, f_{zi}]^T$. Initial compression of the spring for the contact force $\boldsymbol{f}_i$ is denoted by $\boldsymbol{\delta}_{oi} = [\delta_{xoi}, \delta_{yoi}, \delta_{zoi}]^T$.

$$\boldsymbol{f}_i = K_i\boldsymbol{\delta}_{oi}, \quad K_i := \mathrm{diag}[k_{xi}, k_{yi}, k_{zi}] \qquad (11)$$

The potential energy stored in the grasp system is given by

$$U(\boldsymbol{\epsilon}) := \frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{\delta}_{oi} + \boldsymbol{\delta}_i)^T K_i(\boldsymbol{\delta}_{oi} + \boldsymbol{\delta}_i). \qquad (12)$$

Performing Taylor's expansion, the function $U(\boldsymbol{\epsilon})$ can be expressed as

$$U(\boldsymbol{\epsilon}) = U(0) + \boldsymbol{\epsilon}^T G + \frac{1}{2}\boldsymbol{\epsilon}^T H\boldsymbol{\epsilon} + \cdots, \qquad (13)$$

where $G$ and $H$ are the gradient and the hessian, respectively. The grasp system is stable at the initial configuration if and only if the potential energy reaches a local minimum. The function $U(\boldsymbol{\epsilon})$ reaches a local minimum if the following two conditions are satisfied.

(i) $G = 0$,
(ii) $H$ is positive difinite.

Condition (i) is satisfied by Assumption (A2). Hence, we can evaluate grasp stability by eigenvalues of the hessian. The hessian is so-called a grasp stiffness matrix.

Let us define $\nabla := \partial/\partial\boldsymbol{\epsilon}$, and we have the following forms: $G = \nabla U|_0$, $H = \nabla\nabla^T U|_0$. From Eq. (12), the hessian $H$ is given by

$$\begin{aligned} H =& \sum_{i=1}^{n} H_i, \\ H_i :=& k_{xi}(\nabla\delta_{xi}|_0)(\nabla\delta_{xi}|_0)^T + f_{xi}(\nabla\nabla^T\delta_{xi}|_0) \\ &+ k_{yi}(\nabla\delta_{yi}|_0)(\nabla\delta_{yi}|_0)^T + f_{yi}(\nabla\nabla^T\delta_{yi}|_0) \\ &+ k_{zi}(\nabla\delta_{zi}|_0)(\nabla\delta_{zi}|_0)^T + f_{zi}(\nabla\nabla^T\delta_{zi}|_0), \qquad (14)\end{aligned}$$

where, from Eq. (10),

$$\begin{aligned} \nabla\delta_{xi}|_0 &= \begin{bmatrix}\boldsymbol{r}_{xi}\\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{xi}\end{bmatrix}, \\ \nabla\delta_{yi}|_0 &= \begin{bmatrix}\boldsymbol{r}_{yi}\\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} + \kappa_{fi}^{-1}\boldsymbol{r}_{zi}\end{bmatrix} + \frac{\kappa_{fi} + \kappa_{bi}}{\kappa_{fi}}(\nabla v_{ci}|_0), \\ \nabla\delta_{zi}|_0 &= \begin{bmatrix}\boldsymbol{r}_{zi}\\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} - \kappa_{fi}^{-1}\boldsymbol{r}_{yi}\end{bmatrix} + \frac{\kappa_{fi} + \kappa_{ai}}{\kappa_{fi}}(\nabla u_{ci}|_0).\end{aligned}$$

$\nabla\nabla^T\delta_{xi}|_0$, $\nabla\nabla^T\delta_{yi}|_0$, $\nabla\nabla^T\delta_{zi}|_0$ are omitted because they have long terms. In order to derive the hessian, $\nabla u_{ci}|_0$ and $\nabla v_{ci}|_0$ must be derived. These derivatives depend on whether friction exists or not.

# 3 Grasp Stability

## 3.1 Frictionless Grasp

In frictionless case, $f_{yi} = f_{zi} = 0$ because no tangential force occurs. Contact force generated by the displacement of the fingertip is given by $\boldsymbol{f}_i^{'} = \boldsymbol{f}_i + K_i \boldsymbol{\delta}_i$. In frictionless case, the direction of the contact force $\boldsymbol{f}_i^{'}$ is always along the normal direction at current contact point. So we have the following conditions about contact force.

$$R_{fi}(\boldsymbol{f}_i + K_i\boldsymbol{\delta}_i) \perp R_o\boldsymbol{c}_{oiu},\ R_{fi}(\boldsymbol{f}_i + K_i\boldsymbol{\delta}_i) \perp R_o\boldsymbol{c}_{oiv},$$

where $\boldsymbol{c}_{oiu} = \partial\boldsymbol{c}_{oi}/\partial u_{ci}$ and $\boldsymbol{c}_{oiv} = \partial\boldsymbol{c}_{oi}/\partial v_{ci}$ are tangential vectors at the contact point. Hence, we have

$$\begin{aligned} h_{ui}(\boldsymbol{\epsilon}, \boldsymbol{u}_{ci}) &= (R_o\boldsymbol{c}_{oiu})^T\{R_{fi}(\boldsymbol{f}_i + K_i\boldsymbol{\delta}_i)\} = 0, \\ h_{vi}(\boldsymbol{\epsilon}, \boldsymbol{u}_{ci}) &= (R_o\boldsymbol{c}_{oiv})^T\{R_{fi}(\boldsymbol{f}_i + K_i\boldsymbol{\delta}_i)\} = 0. \end{aligned} \tag{15}$$

From the derivation of Appendix A, the hessian $H$ is given by

$$H^s = \sum_{i=1}^{n}(H^s_{fi} + H^s_{ci} + H^s_{kxi} + H^s_{kti}), \tag{16}$$

where the superscript "s" means frictionless contact (i.e. slip or slide). $H^s_{fi}$ expresses an effect of the initial contact force $f_{xi}$.

$$H^s_{fi} = f_{xi}\begin{bmatrix} 0_{3\times3} & [\boldsymbol{r}_{xi}\otimes]^T \\ [\boldsymbol{r}_{xi}\otimes] & \frac{1}{2}([\boldsymbol{c}_i\otimes][\boldsymbol{r}_{xi}\otimes]^T + [\boldsymbol{r}_{xi}\otimes][\boldsymbol{c}_i\otimes]^T) \end{bmatrix} \tag{17}$$

$H^s_{ci}$ means an effect of the local curvatures at the contact, $\kappa_{ai}$, $\kappa_{bi}$, and $\kappa_{fi}$. $H_c \to 0$ if $\kappa_{ai} \to 0$, $\kappa_{bi} \to 0$, $\kappa_{fi} \to \infty$.

$$\begin{aligned} H^s_{ci} = &- f_{xi}\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}} \\ &\times \begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi} + \kappa_{fi}^{-1}\boldsymbol{r}_{zi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi} + \kappa_{fi}^{-1}\boldsymbol{r}_{zi} \end{bmatrix}^T \\ &- f_{xi}\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}} \\ &\times \begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi} - \kappa_{fi}^{-1}\boldsymbol{r}_{yi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi} - \kappa_{fi}^{-1}\boldsymbol{r}_{yi} \end{bmatrix}^T \\ &+ \frac{f_{xi}}{\kappa_{fi}}\begin{bmatrix} 0 & 0 \\ 0 & \boldsymbol{r}_{yi}\boldsymbol{r}_{yi}^T + \boldsymbol{r}_{zi}\boldsymbol{r}_{zi}^T \end{bmatrix} \end{aligned} \tag{18}$$

$H^s_{kxi}$ is an effect of normal stiffness $k_{xi}$.

$$H^s_{kxi} = k_{xi}\begin{bmatrix} \boldsymbol{r}_{xi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{xi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{xi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{xi} \end{bmatrix}^T \tag{19}$$

$H^s_{kti}$ is an effect of tangential stiffness $k_{yi}$ and $k_{zi}$. $H^s_{kti} \to 0$ if $k_{yi} \to \infty$ and $k_{zi} \to \infty$.

$$\begin{aligned} H^s_{kti} = &- \frac{1}{k_{yi} - f_{xi}\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}}\left(f_{xi}\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}\right)^2 \\ &\times \begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi} - \kappa_{bi}^{-1}\boldsymbol{r}_{zi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi} - \kappa_{bi}^{-1}\boldsymbol{r}_{zi} \end{bmatrix}^T \\ &- \frac{1}{k_{zi} - f_{xi}\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}}\left(f_{xi}\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}\right)^2 \\ &\times \begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi} + \kappa_{ai}^{-1}\boldsymbol{r}_{yi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi} + \kappa_{ai}^{-1}\boldsymbol{r}_{yi} \end{bmatrix}^T \end{aligned} \tag{20}$$

From Appendix B, $k_{yi}$, $k_{zi}$, and $f_{xi}$ must be selected from the following regions for stable grasp.

$$k_{yi} - f_{xi}\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}} > 0 \text{ and } k_{zi} - f_{xi}\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}} > 0 \tag{21}$$

Eq. (21) is satisfied if convex shape ($\kappa_\bullet < 0$) is in contact with concave shape ($\kappa_\bullet > 0$), because of $k_{yi} \geq 0, k_{zi} \geq 0, f_{xi} \geq 0$. If Eq. (21) is satisfied, $H^s_{kti}$ is negative semidifinite ($H^s_{kti} \leq 0$) . Hence $H^s_{kti}$ make unstable grasp. However, stability can be made larger, if stiffness $k_{yi}$ and $k_{zi}$ are stronger.

From Assumption (A4) about no rotation of fingertip, offset of the spring model from contact point does not affect the hessian $H$.

We show the relationship among our result and previous works. Nguyen [14] discussed the case of $\kappa_{fi} \to \infty, \kappa_{ai} \to 0, \kappa_{bi} \to 0, k_{yi} \to \infty, k_{zi} \to \infty$. Hence, result of Ref. [14] is expressed as

$$H^s_{Nguyen} = H^s_f + H^s_{kx}.$$

Rimon and Burdick [15], Howard and Kumar [5], Funahashi et al. [4] treated the case of $k_{yi} \to \infty, k_{zi} \to \infty$. Consequently, their results are expressed as

$$H^s_{Funahashi}, H^s_{Howard}, H^s_{Rimon} = H^s_f + H^s_c + H^s_{kx}.$$

## 3.2 Frictional Grasp

If each finger is in contact with the object with friction, no slip occurs at the contact point. Then we have the following condition of pure rolling.

$$R_o\dot{\boldsymbol{c}}_{oi} = R_{fi}\dot{\boldsymbol{c}}_{fi} \tag{22}$$

Frictional grasp stability is formulated by considering Eqs. (9) and (22). From Appendix C, the hessian $H^r$ is given by

$$H^r = \sum_{i=1}^{n}(H^r_{fi} + H^r_{ci} + H^r_{kxi} + H^r_{kti}), \tag{23}$$

where

$$\begin{aligned}
H^r_{fi} =& f_{xi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \frac{1}{2}(\boldsymbol{c}_i\boldsymbol{r}_{xi}^T + \boldsymbol{r}_{xi}\boldsymbol{c}_i^T) - (\boldsymbol{c}_i^T\boldsymbol{r}_{xi})I_3 \end{bmatrix} \\
&+ f_{yi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \frac{1}{2}(\boldsymbol{c}_i\boldsymbol{r}_{yi}^T + \boldsymbol{r}_{yi}\boldsymbol{c}_i^T) - (\boldsymbol{c}_i^T\boldsymbol{r}_{yi})I_3 \end{bmatrix} \\
&+ f_{zi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \frac{1}{2}(\boldsymbol{c}_i\boldsymbol{r}_{zi}^T + \boldsymbol{r}_{zi}\boldsymbol{c}_i^T) - (\boldsymbol{c}_i^T\boldsymbol{r}_{zi})I_3 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
H^r_{ci} =& f_{xi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \frac{1}{\kappa_{ai}+\kappa_{fi}}\boldsymbol{r}_{yi}\boldsymbol{r}_{yi}^T + \frac{1}{\kappa_{bi}+\kappa_{fi}}\boldsymbol{r}_{zi}\boldsymbol{r}_{zi}^T \end{bmatrix} \\
&+ f_{yi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & -\frac{1}{2(\kappa_{ai}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{yi}^T + \boldsymbol{r}_{yi}\boldsymbol{r}_{xi}^T) \end{bmatrix} \\
&+ f_{zi}\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & -\frac{1}{2(\kappa_{bi}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{zi}^T + \boldsymbol{r}_{zi}\boldsymbol{r}_{xi}^T) \end{bmatrix},
\end{aligned}$$

$$H^r_{kxi} = k_{xi}\begin{bmatrix} \boldsymbol{r}_{xi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{xi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{xi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{xi} \end{bmatrix}^T,$$

$$\begin{aligned}
H^r_{kti} =& k_{yi}\begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} \end{bmatrix}^T \\
&+ k_{zi}\begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} \end{bmatrix}^T.
\end{aligned}$$

### 3.3 Effect of Friction

We make an effect of friction clear by comparing frictionless grasp and frictional one under the same condition. For this reason, let us set as $f_{yi} = f_{zi} = 0$. The difference of stability between frictionless grasp and frictional one is given as

$$H^d := H^r - H^s, \tag{24}$$

where

$$\begin{aligned}
H^d_i =& \frac{k_{yi}^2}{k_{yi} - f_{xi}\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}}\begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} - \frac{\kappa_{fi}f_{xi}}{k_{yi}(\kappa_{bi}+\kappa_{fi})}\boldsymbol{r}_{zi} \end{bmatrix} \\
&\times \begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} - \frac{\kappa_{fi}f_{xi}}{k_{yi}(\kappa_{bi}+\kappa_{fi})}\boldsymbol{r}_{zi} \end{bmatrix}^T \\
&+ \frac{k_{zi}^2}{k_{zi} - f_{xi}\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}}\begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} + \frac{\kappa_{fi}f_{xi}}{k_{zi}(\kappa_{ai}+\kappa_{fi})}\boldsymbol{r}_{yi} \end{bmatrix} \\
&\times \begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} + \frac{\kappa_{fi}f_{xi}}{k_{zi}(\kappa_{ai}+\kappa_{fi})}\boldsymbol{r}_{yi} \end{bmatrix}^T.
\end{aligned} \tag{25}$$

$H^d_i$ is positive semi-difinite if Eq. (21) is satisfied. And $\lambda(H^s) = -\infty$ if Eq. (21) is not satisfied, where $\lambda(\bullet)$ represents eingenvalues of $\bullet$. Therefore, friction enhances grasp stability because frictional grasp stability is greater than frictionless one.

# 4 Numerical Examples

Numerical examples show effects of spring stiffness and contact force on the grasp stability. For a simple example, we consider a symmetric 3-finger grasp such as

$$\begin{aligned}
&k_{xi} = k_x,\ k_{yi} = k_y,\ k_{zi} = k_z,\ f_{xi} = f_x, \\
&\kappa_{ai} = \kappa_a,\ \kappa_{bi} = \kappa_b,\ \kappa_{fi} = \kappa_f,\ i = 1, \cdots, n \\
&\boldsymbol{c}_1 = [l_c, 0, 0]^T, \\
&\boldsymbol{c}_2 = [l_c\cos(2\pi/3), l_c\sin(2\pi/3), 0]^T, \\
&\boldsymbol{c}_2 = [l_c\cos(4\pi/3), l_c\sin(4\pi/3), 0]^T, \\
&R_{f1} = [\boldsymbol{r}_{x1}, \boldsymbol{r}_{y1}, \boldsymbol{r}_{z1}] = R_z(0), \\
&R_{f2} = R_z(2\pi/3),\ R_{f3} = R_z(4\pi/3), \\
&R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\end{aligned}$$

In case of frictionless grasp, the hessian $H_s$ is given as

$$\begin{aligned}
&H^s = \mathrm{diag}[H^s_{11}, H^s_{22}, H^s_{33}, H^s_{44}, H^s_{55}, H^s_{66}], \\
&H^s_{11} = H^s_{22} = -\left(\frac{3}{2}\right)\frac{k_y f_x \frac{\kappa_b\kappa_f}{\kappa_b+\kappa_f}}{k_y - f_x\frac{\kappa_b\kappa_f}{\kappa_b+\kappa_f}} + \frac{3}{2}k_x, \\
&H^s_{33} = -3\frac{k_z f_x \frac{\kappa_a\kappa_f}{\kappa_a+\kappa_f}}{k_z - f_x\frac{\kappa_a\kappa_f}{\kappa_a+\kappa_f}}, \\
&H^s_{44} = H^s_{55} \\
&\quad = \left(\frac{3}{2}\right)\frac{(l_c - \kappa_a^{-1})\{k_z(l_c + \kappa_f^{-1}) - f_x\}f_x\frac{\kappa_a\kappa_f}{\kappa_a+\kappa_f}}{k_z - f_x\frac{\kappa_a\kappa_f}{\kappa_a+\kappa_f}}, \\
&H^s_{66} = 3\frac{(l_c - \kappa_b^{-1})\{k_y(l_c + \kappa_f^{-1}) - f_x\}f_x\frac{\kappa_b\kappa_f}{\kappa_b+\kappa_f}}{k_y - f_x\frac{\kappa_b\kappa_f}{\kappa_b+\kappa_f}}.
\end{aligned} \tag{26}$$

In case of frictional grasp, the hessian $H^r$ is obtained as

$$\begin{aligned}
&H^r = \mathrm{diag}[H^r_{11}, H^r_{22}, H^r_{33}, H^r_{44}, H^r_{55}, H^r_{66}], \\
&H^r_{11} = H^r_{22} = 3(k_x + k_y)/2, \\
&H^r_{33} = 3k_z, \\
&H^r_{44} = H^r_{55} = \frac{3}{2}\left(k_z l_c^2 - f_x l_c + \frac{f_x}{\kappa_a + \kappa_f}\right), \\
&H^r_{66} = 3\left(k_y l_c^2 - f_x l_c + \frac{f_x}{\kappa_b + \kappa_f}\right).
\end{aligned} \tag{27}$$

Note that, in this example, $k_y$ is included in $H_{11}$, $H_{22}$, $H_{66}$ and $k_z$ is included in $H_{33}$, $H_{44}$, $H_{55}$.

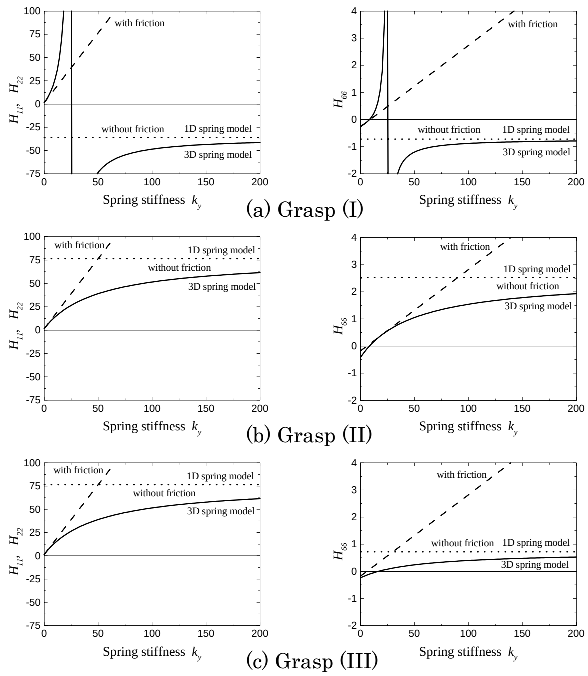



Figure 5: Effect of spring stiffness $k_y$

### 4.1 Effect of Spring Stiffness $k_y$ and $k_z$

Fig.5 shows an effect of spring stiffness $k_y$, $k_z$ on $H_{11}$, $H_{22}$, $H_{66}$ when $k_x = 1.0$, $f_x = 1.0$, $l_c = 0.1$ and

Grasp (I) $\kappa_a = \kappa_b = 1/0.02,\ \kappa_f = 1/0.02,$

Grasp (II) $\kappa_a = \kappa_b = -1/0.04,\ \kappa_f = 1/0.02,$

Grasp (III) $\kappa_a = \kappa_b = 1/0.02,\ \kappa_f = -1/0.04.$

Since $H_{33}$, $H_{44}$, $H_{55}$ are similar to Fig.5, we omit them. In case of Grasp (I), the range of $k_y \leq 25$ or $k_z \leq 25$ does not satisfy Eq. (21). Grasp (I) is unstable for any stiffness of $k_y$ and $k_z$, because both finger and object are convex. Grasp (II) and (III) become unstable if spring stiffness is small. The range of spring stiffness, which makes the hessian positive definiteness and makes the grasp stable, depend on whether friction exists or not. In case of frictionless contact, $k_y$ and $k_z$ must be selected from

$$k_y^s > f_x/(l_c + \kappa_f^{-1}),\ k_z^s > f_x/(l_c + \kappa_f^{-1}).$$

In case of frictional contact, $k_y$ and $k_z$ must be selected from

$$k_y^r > f_x \frac{l_c(\kappa_b + \kappa_f) - 1}{l_c^2(\kappa_b + \kappa_f)},\ k_z^r > f_x \frac{l_c(\kappa_a + \kappa_f) - 1}{l_c^2(\kappa_a + \kappa_f)}.$$

### 4.2 Effect of Contact Force $f_x$

Fig.6 shows an effect of contact force $f_x$ on $H_{11}$, $H_{22}$, and $H_{66}$ of Grasp (II). There exists contact force which makes unstable grasp about rotation. For this reason, we must obtain the range which makes stable grasp. In case of frictionless contact, $f_x$ must be selected from

$$0 < f_x^s < \min\{k_y(l_c + \kappa_f^{-1}),\ k_z(l_c + \kappa_f^{-1})\}. \tag{28}$$

In case of frictional contact, $f_x$ must be selected from

$$0 < f_x^r < \min\{\frac{k_z l_c^2(\kappa_a + \kappa_f)}{l_c(\kappa_a + \kappa_f) - 1}, \frac{k_y l_c^2(\kappa_b + \kappa_f)}{l_c(\kappa_b + \kappa_f) - 1}\}. \tag{29}$$

There exists contact force which maximizes rotational grasp stability. From $\partial H_{44}^s/\partial f_x = 0$ and $\partial H_{66}^s/\partial f_x = 0$, it is given at

$$f_x^s = k_z \left( \kappa_a^{-1} + \kappa_f^{-1} + \kappa_a^{-1} \sqrt{\kappa_f^{-1}(\kappa_a + \kappa_f)(1 - l_c \kappa_a)} \right),$$
$$f_x^s = k_y \left( \kappa_b^{-1} + \kappa_f^{-1} + \kappa_b^{-1} \sqrt{\kappa_f^{-1}(\kappa_b + \kappa_f)(1 - l_c \kappa_b)} \right).$$

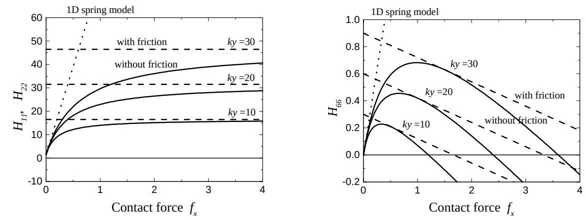



Figure 6: Effect of contact force $f_x$ for Grasp (II)

# 5 Conclusions

This paper discussed stability of 3D grasps from the viewpoint of potential energy method.

We introduced 3D spring model into frictionless 3D grasps in order to obtain accurate grasp stability. The relation between object's displacement and finger's one is derived. It was precisely formulated that a finger generates contact force along the normal direction at current contact point. The hessian $H^s$ is obtained, and the frictionless grasp stability is evaluated by the eigenvalues of $H^s$. The conditions of $k_{yi}$, $k_{zi}$, $f_{xi}$ for stable grasp is also provided. From numerical examples, we showed that there exist the range of contact force for stable grasp and an optimum contact force.

Moreover, in frictional case, the hessian $H^r$ is established by using contact kinematcs of pure rolling. It is proved that friction enhances grasp stability by comparing frictional grasp stability with frictionless one.

If a coefficient of contact friction is unknown, the grasp is assumed to be frictionless one. The grasp is made stable by using the eigenvalues of $H^s$. Then, it is guaranteed that the grasp is stable, whether friction exists or not.

While Kerr and Roth [8] and Nakamura et al. [13] reduced the problem of determining optimum contact force to a linear or nonlinear programming problem, our analysis about frictionless grasp suggested that optimum contact force can be determined from the viewpoint of the grasp stability.

# References


[1] H. Bruyninckx, S. Demey, and V. Kumar, Generalized Stability of Compliant Grasps, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 2396-2402, (1998).

[2] K. K. Choi, S. L. Jiang, and Z. Li, Grasping with Elastic Finger Tips, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 920-925, (1999).

[3] M. R. Cutkosky and I. Kao, Computing and Controlling the Compliance of a Robotic Hand, *IEEE Trans. on Robotics and Automation*, Vol. 5, No. 2, pp. 151-165, (1989).

[4] Y. Funahashi, T. Yamada, M. Tate, and Y. Suzuki, Grasps stability analysis considering the curvatures at contact points, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 3040-3046, (1996).

[5] W. S. Howard and V. Kumar, On the Stability of Grasped Objects, *IEEE Trans. on Robotics and Automation*, Vol. 12, No. 6, pp. 904-917, (1996).

[6] W. S. Howard and V. Kumar, Modeling and Analysis of the Compliance and Stability of Enveloping Grasps, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 1367-1372, (1995).

[7] M. Kaneko, N. Imamura, K. Yokoi, and K. Tanie, A realization of stable grasp based on virtual stiffness model by robot fingers, *Proc. of IEEE Int. Workshop on Advanced Motion Control*, pp. 156-163C(1990).

[8] J. Kerr and B. Roth, Analysis of multifingered hands, *Int. Journal of Robotics Research*, Vol. 4, No. 4, pp. 3-17, (1986).

[9] Q. Lin, J. Burdick, and E. Rimon, A Quality Measure for Compliant Grasps, *Proc. of IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp. 86-92, (1997).

[10] Q. Lin, J. Burdick, and E. Rimon, Computation and Analysis of Compliance in Grasping and Fixturing, *Proc. of IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp. 93-99, (1997).

[11] H. Maekawa, K. Tanie, and K. Komoriya, Kinematics, Statics and Stiffness Effect of 3D Grasp by multifingered hand with rolling contact at the fingertip, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 78-85, (1997).

[12] D. J. Montana, Contact Stability for Two-Fingered Grasps, *IEEE Trans. on Robotics and Automation*, Vol. 8, No. 4, pp. 421-430, (1992).

[13] Y. Nakamura, K. Nagai, and T. Yoshikawa, Dynamics and Stability in Coordination of Multiple Robotic Mechanisms, *Int. Journal of Robotics Research*, Vol. 8, No. 2, pp. 44-61, (1989).

[14] V. D. Nguyen, Constructing stable grasps, *Int. Journal of Robotics Research*, Vol. 7, No. 3, pp. 3-16, (1988).

[15] E. Rimon and J. W. Burdick, Mobility of bodies in contact - Part II: How Forces are Generated by Curvature Effects, *IEEE Trans. of Robotics and Automation*, pp. 709-717, (1998).

[16] S. K. Saha, T. Yamada, N. Mimura, and Y. Funahashi, Stability analysis of frictionless planar grasps with 2D spring model, *Int. Journal of JSME*, Series C, Vol. 42, No. 4, pp. 974-981, (1999).

[17] K. B. Shimoga, Robot grasp synthesis algorithms: A survey. *Int. Journal of Robotics Research*, Vol. 15, No. 3, pp. 230-266, (1996).

[18] T. G. Sugar and V. Kumar, Metrics for Analysis and Optimization of Grasps and Fixtures, *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 3561-3566, (2000).

[19] M. M. Svinin, K. Ueda, and M. Kaneko Analytical Conditions for the Rotational Stability of an Object in Multi-Finger Grasping, *Proc. of IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp. 257-262, (1999).

[20] T. Yamada, S. K. Saha, N. Mimura, and Y. Funahashi, Stability analysis of grasping an arbitrary shaped planar object, *Trans. of JSME*, Series C, Vol. 64, No. 621, pp. 1736-1742, (1998), (in Japanese).

[21] C. H. Xiong, Y. F. Li, H. Ding, and Y. L. Xiong, On the Dynamic Stability of Grasping, *Int. Journal of Robotics Research*, Vol. 18, No. 9, pp. 951-958, (1999).


# Appendix A: Frictionless Grasp

From Eq. (15), we have

$$\nabla h_{ui} + \frac{\partial h_{ui}}{\partial u_{ci}} \nabla u_{ci} + \frac{\partial h_{ui}}{\partial v_{ci}} \nabla v_{ci} = 0,$$
$$\nabla h_{vi} + \frac{\partial h_{vi}}{\partial u_{ci}} \nabla u_{ci} + \frac{\partial h_{vi}}{\partial v_{ci}} \nabla v_{ci} = 0.$$

Then $\nabla u_{ci}$ and $\nabla v_{ci}$ are formulated as

$$[\, \nabla u_{ci} \quad \nabla v_{ci} \,] = -[\, \nabla h_{ui} \quad \nabla h_{vi} \,] \begin{bmatrix} \frac{\partial h_{ui}}{\partial u_{ci}} & \frac{\partial h_{vi}}{\partial u_{ci}} \\ \frac{\partial h_{ui}}{\partial v_{ci}} & \frac{\partial h_{vi}}{\partial v_{ci}} \end{bmatrix}^{-1}.$$

Considering initial conditions yields

$$\nabla u_{ci}|_0 = \frac{1}{\kappa_{ai}\kappa_{fi} f_{xi} - (\kappa_{ai} + \kappa_{fi}) k_{zi}} \times \left[ \kappa_{fi} k_{zi} \begin{bmatrix} \boldsymbol{r}_{zi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{zi} \end{bmatrix} + (\kappa_{fi} f_{xi} - k_{zi}) \begin{bmatrix} 0 \\ \boldsymbol{r}_{yi} \end{bmatrix} \right],$$
$$\nabla v_{ci}|_0 = \frac{1}{\kappa_{bi}\kappa_{fi} f_{xi} - (\kappa_{bi} + \kappa_{fi}) k_{yi}} \times \left[ \kappa_{fi} k_{yi} \begin{bmatrix} \boldsymbol{r}_{yi} \\ \boldsymbol{c}_i \otimes \boldsymbol{r}_{yi} \end{bmatrix} - (\kappa_{fi} f_{xi} - k_{yi}) \begin{bmatrix} 0 \\ \boldsymbol{r}_{zi} \end{bmatrix} \right].$$

Substituting the above equations into $\nabla\delta_{yi}|_0$ and $\nabla\delta_{zi}|_0$ yields

$$\nabla\delta_{yi}|_0 = -\frac{\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}f_{xi}}{k_{yi}-\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}f_{xi}}\begin{bmatrix}\boldsymbol{r}_{yi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi}-\frac{1}{\kappa_{bi}}\boldsymbol{r}_{zi}\end{bmatrix},$$

$$\nabla\delta_{zi}|_0 = -\frac{\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}f_{xi}}{k_{zi}-\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}f_{xi}}\begin{bmatrix}\boldsymbol{r}_{zi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi}+\frac{1}{\kappa_{ai}}\boldsymbol{r}_{yi}\end{bmatrix}.$$

In a similar way, we have

$$\begin{aligned}\nabla\nabla^T\delta_{xi}|_0 &= \begin{bmatrix}0 & 0\\ 0 & \frac{1}{2}([\boldsymbol{c}_i\otimes][\boldsymbol{r}_{xi}\otimes]+[\boldsymbol{r}_{xi}\otimes][\boldsymbol{c}_i\otimes])\end{bmatrix}\\
&+\frac{1}{\kappa_{bi}}\begin{bmatrix}0\\ \boldsymbol{r}_{zi}\end{bmatrix}\begin{bmatrix}0\\ \boldsymbol{r}_{zi}\end{bmatrix}^T+\frac{1}{\kappa_{ai}}\begin{bmatrix}0\\ \boldsymbol{r}_{yi}\end{bmatrix}\begin{bmatrix}0\\ \boldsymbol{r}_{yi}\end{bmatrix}^T\\
&-\frac{\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}k_{yi}^2}{(k_{yi}-\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}f_{xi})^2}\\
&\times\begin{bmatrix}\boldsymbol{r}_{yi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi}-\frac{1}{\kappa_{bi}}\boldsymbol{r}_{zi}\end{bmatrix}\begin{bmatrix}\boldsymbol{r}_{yi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi}-\frac{1}{\kappa_{bi}}\boldsymbol{r}_{zi}\end{bmatrix}^T\\
&-\frac{\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}k_{zi}^2}{(k_{zi}-\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}f_{xi})^2}\\
&\times\begin{bmatrix}\boldsymbol{r}_{zi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi}+\frac{1}{\kappa_{ai}}\boldsymbol{r}_{yi}\end{bmatrix}\begin{bmatrix}\boldsymbol{r}_{zi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi}+\frac{1}{\kappa_{ai}}\boldsymbol{r}_{yi}\end{bmatrix}^T\end{aligned}$$

# Appendix B: Eq. (21)

The term $\frac{\partial h_{ui}}{\partial u_{ci}}$ represents relationship between contact position and contact force. This relation is illustrated in Fig. 7. Hence, we have the following three conditions.

$$\text{(a) if } \kappa_a > 0, \kappa_f > 0 \text{ then } \left.\frac{\partial h_{ui}}{\partial u_{ci}}\right|_0 > 0,$$

$$\text{(b) if } \kappa_a < 0, \kappa_f > 0 \text{ then } \left.\frac{\partial h_{ui}}{\partial u_{ci}}\right|_0 > 0,$$

$$\text{(c) if } \kappa_a > 0, \kappa_f < 0 \text{ then } \left.\frac{\partial h_{ui}}{\partial u_{ci}}\right|_0 < 0,$$

where

$$\left.\frac{\partial h_{ui}}{\partial u_{ci}}\right|_0 = \kappa_{fi}(\kappa_{fi}+\kappa_{ai})\left\{k_{zi}-\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}f_{xi}\right\}.$$

From these conditions, $k_{zi}$ and $f_{xi}$ are restricted to

$$k_{zi}-\frac{\kappa_{ai}\kappa_{fi}}{\kappa_{ai}+\kappa_{fi}}f_{xi} > 0. \tag{30}$$

In a similar way, from $\frac{\partial h_{vi}}{\partial v_{ci}}$, $k_{yi}$ and $f_{xi}$ are restricted to

$$k_{yi}-\frac{\kappa_{bi}\kappa_{fi}}{\kappa_{bi}+\kappa_{fi}}f_{xi} > 0. \tag{31}$$

If Eqs. (30) and/or (31) are not satisfied, the grasp is unstable, and eigenvalues of the hessian $H_{kti}^s$ are infinity ($\lambda(H_{kti}^s) = -\infty$).

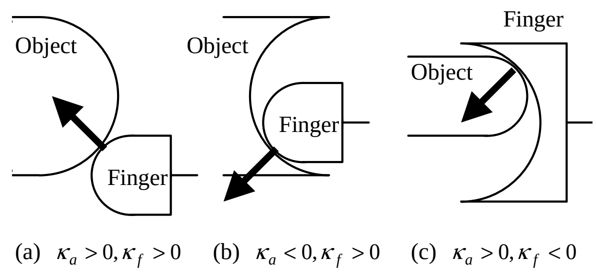



Figure 7: Relationship between finger's motion and contact force

# Appendix C: Frictional Grasp

In frictional case, the first and the second derivatives of $u_{ci}$, $v_{ci}$ is given by

$$\nabla u_{ci} = \begin{bmatrix}\mathbf{0}_3\\ \frac{1}{\kappa_{ai}+\kappa_{fi}}\boldsymbol{r}_{yi}\end{bmatrix}, \nabla v_{ci} = \begin{bmatrix}\mathbf{0}_3\\ -\frac{1}{\kappa_{bi}+\kappa_{fi}}\boldsymbol{r}_{zi}\end{bmatrix},$$

$$\nabla\nabla^T u_{ci} = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{0}_{3\times3}\\ \mathbf{0}_{3\times3} & \frac{\kappa_{bi}}{2(\kappa_{ai}+\kappa_{fi})(\kappa_{bi}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{zi}^T+\boldsymbol{r}_{zi}\boldsymbol{r}_{xi}^T)\end{bmatrix},$$

$$\nabla\nabla^T v_{ci} = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{0}_{3\times3}\\ \mathbf{0}_{3\times3} & \frac{\kappa_{ai}}{2(\kappa_{ai}+\kappa_{fi})(\kappa_{bi}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{yi}^T+\boldsymbol{r}_{yi}\boldsymbol{r}_{xi}^T)\end{bmatrix}.$$

Then the derivative of $\delta_{xi}$, $\delta_{yi}$, and $\delta_{zi}$ are derived as

$$\nabla\delta_{yi}|_0 = \begin{bmatrix}\boldsymbol{r}_{yi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{yi}\end{bmatrix},\ \nabla\delta_{zi}|_0 = \begin{bmatrix}\boldsymbol{r}_{zi}\\ \boldsymbol{c}_i\otimes\boldsymbol{r}_{zi}\end{bmatrix},$$

$$\begin{aligned}\nabla_\xi\nabla_\xi^T\delta_{xi}|_0 &= \frac{1}{\kappa_{ai}+\kappa_{fi}}\boldsymbol{r}_{yi}\boldsymbol{r}_{yi}^T+\frac{1}{\kappa_{bi}+\kappa_{fi}}\boldsymbol{r}_{zi}\boldsymbol{r}_{zi}^T\\
&-(\boldsymbol{c}_i^T\boldsymbol{r}_{xi})I_3+\frac{1}{2}(\boldsymbol{c}_i\boldsymbol{r}_{xi}^T+\boldsymbol{r}_{xi}\boldsymbol{c}_i^T),\\
\nabla_\xi\nabla_\xi^T\delta_{yi}|_0 &= -\frac{1}{2(\kappa_{ai}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{yi}^T+\boldsymbol{r}_{yi}\boldsymbol{r}_{xi}^T)\\
&-(\boldsymbol{c}_i^T\boldsymbol{r}_{yi})I_3+\frac{1}{2}(\boldsymbol{c}_i\boldsymbol{r}_{yi}^T+\boldsymbol{r}_{yi}\boldsymbol{c}_i^T),\\
\nabla_\xi\nabla_\xi^T\delta_{zi}|_0 &= -\frac{1}{2(\kappa_{bi}+\kappa_{fi})}(\boldsymbol{r}_{xi}\boldsymbol{r}_{zi}^T+\boldsymbol{r}_{zi}\boldsymbol{r}_{xi}^T)\\
&-(\boldsymbol{c}_i^T\boldsymbol{r}_{zi})I_3+\frac{1}{2}(\boldsymbol{c}_{oi}\boldsymbol{r}_{zi}^T+\boldsymbol{r}_{zi}\boldsymbol{c}_{oi}^T).\end{aligned}$$