Grasp Stability Analysis Considering the Curvatures at Contact Points

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Abstract

We propose a new method for analyzing the stability of grasps. The characteristic points in this paper are to consider the curvature of both hand and object at contact points and the grasp with friction and frictionless contact. From this analysis, it is shown that the grasp using round fingers is more stable than using sharp fingers. Moreover, we establish the condition on the finger's stiffness to stabilize the grasp with friction. It is proved that the required stiffness of fingers are decreased by considering the curvature. The stability analysis is greatly simplified by using potential energy of the grasp system and is of practical use.

1. Introduction

The grasp stability of a robot hand is an important problem. The grasp system is required to maintain the contact state against any external disturbances applied to the object.

The stability of static grasp is analyzed by the concept of form-closure and force-closure grasps [1-5]. The form-closure grasp geometrically restricts the object by fingers of the hand, an object displacement due to external disturbances does not occur. So the form-closure grasp does not break. The force-closure grasp can generate any force and moment by finger forces only. So the grasp can resist any external disturbances, and be maintained. These types of grasp are stable and any displacement of the object does not occur in spite of external forces.

On the other hand, many investigations have been performed on the grasp stability allowing an object displacement due to external disturbances. These works discuss the stability from the viewpoint of either potential energy or restoring force by replacing the hand and object configuration with an elastic system. These works are classified into two categories whether each finger slides along the object surface or not.

Hanafusa and Asada [6] investigated fingertip positions for stabilizing frictionless grasps. It was shown that the grasp is in stable configuration when the potential energy of the grasp system is minimized. Nguyen [7] discussed the stability of the grasp at an equilibrium state when each finger's stiffness is controlled. The potential energy was derived with the curvature of the object taken into consideration. It was shown that the grasp is in stable configuration if and only if the potential energy is locally minimum at the equilibrium state. It was also shown how the object's curvature influences the grasp stability.

Kaneko et al. [8] analyzed planar grasps with friction, and gave the condition on finger's stiffness to stabilize the grasp. Mimura and Funahashi [9] extended the planar grasps to spatial grasps. However, the influence of the curvature on the grasp stability is not investigated because the object is held by sharp fingers.

D. J. Montana [10,11] analyzed the grasp stability considering the curvatures of both the hand and the object. It is assumed that the object is grasped by two fingers, and the magnitude of each finger force exerted at contact point is constant. Howard and Kumar [12] studied the grasp stability from the viewpoint of restoring force. However, the analysis of forces generated at contact points is somewhat complex.

This paper analyzes the stability of planar grasps by considering the curvature of both object and fingers at contact points as shown in Fig. 1. The influences of the curvature on stability of the grasps with friction and frictionless point contact are investigated. First, fingers of the hand are replaced by elastic fingers whose curvature is taken into consideration, and a potential function of this grasping system is derived. Then the stability is evaluated by using the second-order partial derivatives of the potential function. Moreover, we analyze the condition on the finger's stiffness to stabilize the grasp with friction. The stability analysis is greatly simplified by using potential energy of the grasp system and is of practical use.

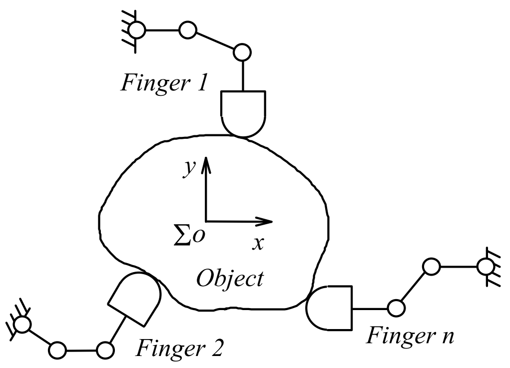


Fig. 1 A planar grasp considering the curvature

2. Stability of frictionless grasps

2.1. Modeling of grasps

In this section, we discuss the grasp stability considering the curvature of an object and fingers when the fingers slide on the object surface without friction. Because of frictionless, each finger force is along the inward normal of object at contact point.

We define coordinate frames shown in Fig. 2. The origin of the object coordinate frame Σ_o is fixed at arbitrary position on the object. The object is approximated by a circle at the contact point as shown in Fig. 2. The origin of the contact-point coordinate frame Σ_i is fixed at the center of curvature, and the axis x_i of Σ_i is along outward normal of object at contact point. The relative position of the origin of Σ_i with respect to Σ_o is denoted by $\boldsymbol{p}_i$, the relative orientation of Σ_i with respect to Σ_o is denoted by θ_i. The radius of curvature of the object and the finger at i-th contact point is defined by R_i and r_i, respectively. If the object has convex arc at the i-th contact point as shown in Fig. 3(a), we have $R_i > 0$. If the object has concave arc as shown in Fig. 3(b), we have $R_i < 0$. The radius r_i is defined in a similar way.

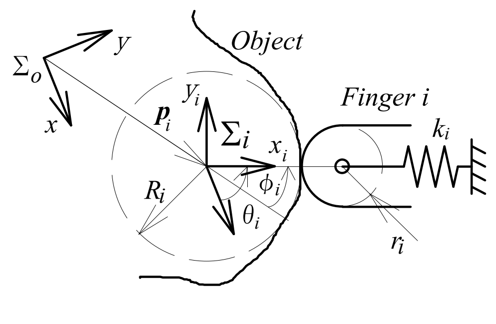
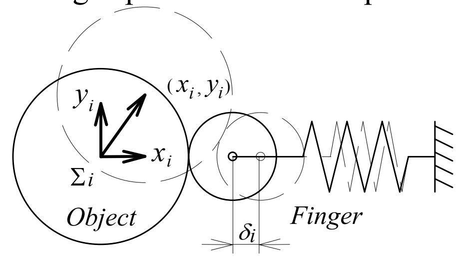
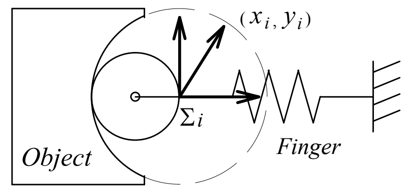


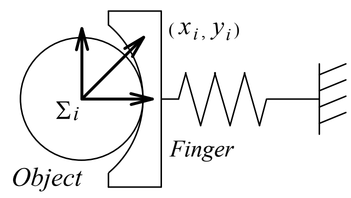
Fig. 2 A grasp with frictionless point contact



(a) Convex finger and convex object



(b) convex finger and concave object



(c) concave finger and convex object

Fig. 3 Compression due to object displacement

In order to simplify the discussion, we make following assumptions.

(2.1) Each finger is in a frictionless point contact with the object and has a virtual spring shown in Fig. 2.

(2.2) The shapes of an object and each finger are known, and can be approximated by circles of the curvature, respectively.

(2.3) The grasping system is in an equilibrium state initially, and contact points and contact forces are known.

(2.4) Two dimensional grasp.

By setting virtual springs as assumption (2.1), we can derive potential energy of the grasp system. Since the shape of the object is represented by the circles of curvature up to second order exactly, the second-order partial derivatives of the potential function is exactly derived according to assumption (2.2)

We will derive the potential energy stored in the grasp system due to external disturbance and investigate grasp stability at the equilibrium state given by assumption (2.3). We consider the relation between object displacement and compression of virtual springs. In order to make this analysis clear, the relation between displacement in Σ_i and the compression is studied firstly, then the relation between displacement in Σ_o and the compression is investigated.

2.2. Compression of virtual springs

2.2.1. Contact-point Coordinate Frame

In this section, we will formulate the compression δ_i at the virtual spring k_i when infinitesimal translation (x_i, y_i) and rotation ζ_i at the origin of Σ_i occur. Suppose that both the object and the i-th finger have convex arcs around the contact point as shown in Fig. 3(a).

Let us define $(A_i, B_i)^T$ and $(a_i, b_i)^T$ as the center position of the object's and the i-th finger's curvature around the i-th contact point respectively. From the definition of the i-th contact coordinate frame Σ_i, we have

$$(A_i, B_i)^T = (0,0)^T,$$
$$(a_i, b_i)^T = (R_i + r_i, 0)^T. \qquad (1)$$

With the displacement of (x_i, y_i, ζ_i), the virtual spring k_i is compressed. So we have

$$(A_i, B_i)^T = (x_i, y_i)^T,$$
$$(a_i, b_i)^T = (R_i + r_i + \delta_i, 0)^T. \qquad (2)$$

When the i-th finger maintains in contact with the object, the distance between the center of two circles is equal to the sum of the radii R_i and r_i. Hence, the following relation holds.

$$(A_i - a_i)^2 + (B_i - b_i)^2 = (R_i + r_i)^2. \qquad (3)$$

By substituting Eq. (2) into Eq. (3), the compression δ_i is given by

$$\delta_i = x_i - (R_i + r_i) \pm \sqrt{(R_i + r_i)^2 - {y_i}^2}. \qquad (4)$$

Considering the physical constraint, we have

$$\delta_i = x_i - (R_i + r_i) + \sqrt{(R_i + r_i)^2 - {y_i}^2}. \qquad (5)$$

Equation (5) denotes the relationship between the displacement (x_i, y_i, ζ_i) with respect to Σ_i and the compression δ_i at the virtual spring k_i.

In the same way, when either the object or the i-th finger has concave arc around the i-th contact point as shown in Fig. 3(b), (c), the compression δ_i is given by

$$\delta_i = x_i - (R_i + r_i) - \sqrt{(R_i + r_i)^2 - {y_i}^2}. \qquad (6)$$

2.2.2. Object Coordinate Frame

We investigate the relationship between the displacement at the origin of Σ_o and the compression δ_i. When infinitesimal translation (x, y) and rotation ζ occur at the origin of Σ_o due to external disturbance, the displacement at $\boldsymbol{p}_i$, which is the position of the origin of Σ_i, is represented by

$$(x, y)^T + \left(\mathrm{Rot}(\zeta) - I_2\right)\boldsymbol{p}_i, \qquad (7)$$

where I_2 is a 2×2 identity matrix, and $\mathrm{Rot}(\bullet)$ is a rotation matrix represented by

$$\mathrm{Rot}(\bullet) = \begin{bmatrix} \cos(\bullet) & -\sin(\bullet) \\ \sin(\bullet) & \cos(\bullet) \end{bmatrix}.$$

By transforming the displacement (x, y, ζ) in Σ_o into the displacement (x_i, y_i, ζ_i) in Σ_i, we have

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \mathrm{Rot}(-\theta_i)\left\{\begin{pmatrix} x \\ y \end{pmatrix} + \left(Rot(\zeta) - I_2\right)\boldsymbol{p}_i\right\}, \qquad (8)$$

$$\zeta_i = \zeta \qquad (9)$$

Using

$$\boldsymbol{p}_i = p_i\left(\cos(\theta_i - \phi_i), \sin(\theta_i - \phi_i)\right)^T,$$

we have

$$x_i = x\cos\theta_i + y\sin\theta_i + p_i\{\cos(\zeta - \phi_i) - \cos\phi_i\},$$
$$y_i = -x\sin\theta_i + y\cos\theta_i + p_i\{\sin(\zeta - \phi_i) + \sin\phi_i\}. (10)$$

Substituting Eq. (10) into Eq. (5) yields the relation between the object displacement (x, y, ζ) at the origin of Σ_o and the compression δ_i at the spring k_i.

2.3. Evaluation of grasp stability

In this section, we will discuss the stability of the equilibrium state by using the potential function of the grasp system. Since the potential energy U stored in the whole grasp system is the sum of the energy U_i stored in the i-th spring, we have

$$U = \sum U_i = \sum \tfrac{1}{2} k_i (\delta_i + \delta_{oi})^2, \qquad (11)$$

where δ_{oi} is an initial compression of the i-th spring.

The grasp is in stable configuration if and only if the potential energy U is locally minimum at the equilibrium state. The Taylor expansion of U about the equilibrium state is given by

$$U = U(0) + \boldsymbol{\varepsilon}^T \nabla U\big|_{(0)} + \tfrac{1}{2}\boldsymbol{\varepsilon}^T H\big|_{(0)}\boldsymbol{\varepsilon} + \cdots, \qquad (12)$$

where

$$\boldsymbol{\varepsilon} = \begin{pmatrix} x \\ y \\ \zeta \end{pmatrix}, \quad \nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial \zeta} \end{pmatrix}, \quad H = \begin{bmatrix} \frac{\partial^2 U}{\partial x^2} & \frac{\partial^2 U}{\partial x \partial y} & \frac{\partial^2 U}{\partial x \partial \zeta} \\ \frac{\partial^2 U}{\partial y \partial x} & \frac{\partial^2 U}{\partial y^2} & \frac{\partial^2 U}{\partial y \partial \zeta} \\ \frac{\partial^2 U}{\partial \zeta \partial x} & \frac{\partial^2 U}{\partial \zeta \partial y} & \frac{\partial^2 U}{\partial \zeta^2} \end{bmatrix}$$

From Eq. (13), the potential function U is locally minimum at the equilibrium state if and only if the following two conditions are satisfied.

(1) $\nabla U\big|_{(0)} = 0$

(2) $H|_{(0)}$ is positive definite

By assumption (2.3), condition (1) is always satisfied.

From Eqs. (6), (10) and (11), elements of $H|_{(0)}$ can be estimated

$$H|_{(0)} = \begin{bmatrix} k_{xx} & k_{xy} & k_{x\zeta} \\ k_{yx} & k_{yy} & k_{y\zeta} \\ k_{\zeta x} & k_{\zeta y} & k_{\zeta\zeta} \end{bmatrix} \tag{13}$$

$$k_{xx} = \sum (k_i c_i^2 - \frac{f_i}{R_i + r_i} s_i^2) ,$$

$$k_{xy} = k_{yx} = \sum (k_i + \frac{f_i}{R_i + r_i}) c_i s_i ,$$

$$k_{x\zeta} = k_{\zeta x} = \sum \{ l_{yi} k_i c_i + (l_{xi} - R_i) \frac{f_i}{R_i + r_i} s_i \} ,$$

$$k_{yy} = \sum (k_i s_i^2 - \frac{f_i}{R_i + r_i} c_i^2) ,$$

$$k_{y\zeta} = k_{\zeta y} = \sum \{ l_{yi} k_i s_i - (l_{xi} - R_i) \frac{f_i}{R_i + r_i} c_i \} ,$$

$$k_{\zeta\zeta} = \sum \{ l_{yi}^2 k_i - (l_{xi} - R_i)(l_{xi} + r_i) \frac{f_i}{R_i + r_i} \} \tag{14}$$

where

$$l_{xi} = p_i \cos\phi_i + R_i ,\ \ l_{yi} = p_i \sin\phi_i ,$$
$$c_i = \cos\theta_i ,\ \ s_i = \sin\theta_i ,$$
$$f_i = k_i \delta_{oi} .$$

$f_i = k_i \delta_{oi}$ means an initial grasping force of the i-th finger.

k_i and f_i can be obtained from stiffness constant of virtual spring and initial grasping force, respectively. The object's curvature R_i, the finger's curvature r_i, and the relative position $\boldsymbol{p}_i$ is known by assumptions (2.2) and (2.3). If all the eigenvalues of $H|_{(0)}$ are positive, the grasp is in stable configuration.

2.4. Examples

In this section, we investigate the influence of the curvatures on the stability of grasps shown in Fig. 4 by using the proposed method.

Case (a) - (e) are simple examples of two-finger grasp with equal springs. In these cases, since we have

$$\theta_1 = 0 ,\ \ \theta_2 = \pi ,$$
$$r_1 = r_2 = r ,\ \ R_1 = R_2 = R ,$$
$$k_1 = k_2 = k > 0 ,\ \ f_1 = f_2 = f > 0 ,$$
$$l_{x1} = l_{x2} = l > 0 ,\ \ l_{y1} = l_{y2} = 0 ,$$
$$p_1 = p_2 = p > 0 ,$$

elements of $H|_{(0)}$ become

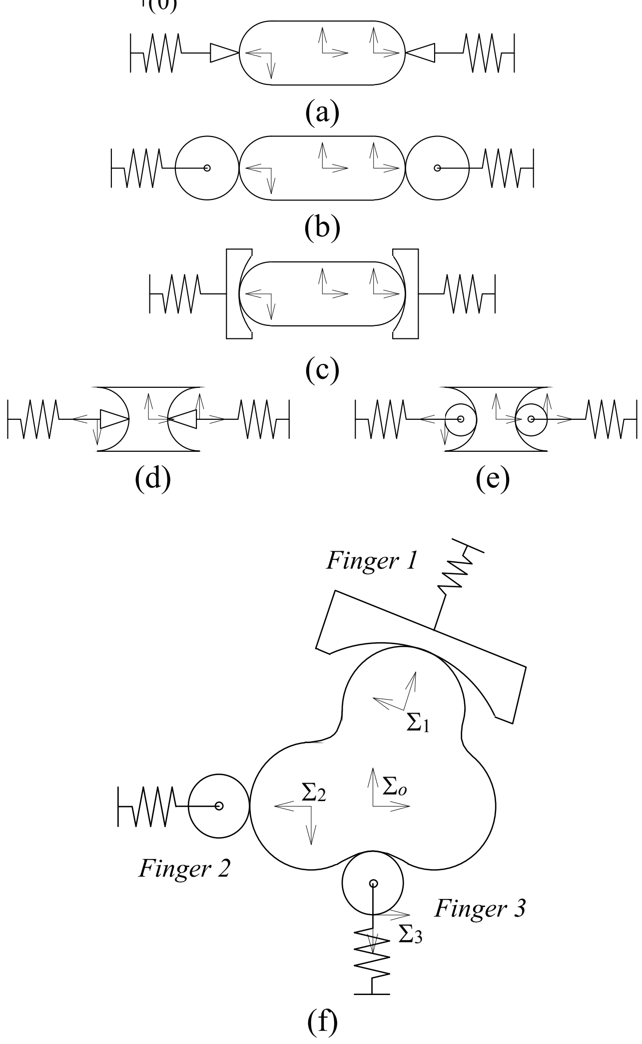


Fig. 4 Stability of frictionless grasps

$$k_{xx} = 2k > 0 ,$$
$$k_{yy} = -2f/(R + r) ,$$
$$k_{\zeta\zeta} = -2f(l_x - R)(l_x + r)/(R + r) ,$$
$$k_{xy} = k_{yx} = k_{x\zeta} = k_{\zeta x} = k_{y\zeta} = k_{\zeta y} = 0 ,$$

where $l = p + R$ means the distance between the origin of Σ_o and the contact point. k_{xx}, k_{yy} and $k_{\zeta\zeta}$ imply the eigenvalues of $H|_{(0)}$. Hence, we get the following results. The grasp is always unstable if a convex object is held by convex fingers as shown in Fig. 4(a)(b). The grasp is stable if a convex object is held by fingers with $-l < r < -R$ as shown in Fig. 4(c). The grasp is always stable if a concave object is held as shown in Fig. 4(d)(e). Moreover, it is obtained that the larger the radius *r* is, the larger the eigenvalues are. So, the grasp using round fingers is more stable than using sharp fingers.

Similar results are also provided in Refs. [3] - [5] from viewpoint of the form-closure.

Case (f) is an example of three-finger grasp. Table 1 shows the parameters and the eigenvalues of $H|_{(0)}$. The stability of the grasp cannot be determined by inspection. However, the grasp is stable because all the eigenvalues are positive.

3. Stability of grasps with friction

3.1. Modeling

In this section, we consider stability of grasps with friction. In order to simplify the discussion, we make the following assumptions.

(3.1) Finger and object are in contact with friction.

(3.2) Each fingertip does not rotate because angular stiffness of the finger is strong enough.

(3.3) An equilibrium grasp is given initially.

By assumption (3.1), each finger does not slide on the object surface. Note that the assumption (3.1) is the key difference from section 2.

We use the same coordinate frames Σ_o and Σ_i defined in section 2. A spring along the axis y_i is required because each finger can generate tangential force at the contact point. So, the virtual spring k_i is replaced by two virtual springs k_{xi} and k_{yi}, which are fixed at the center of curvature of the i-th contact point as shown in Fig. 5. k_{xi} and k_{yi} are stiffness constants along x_i and y_i axis.

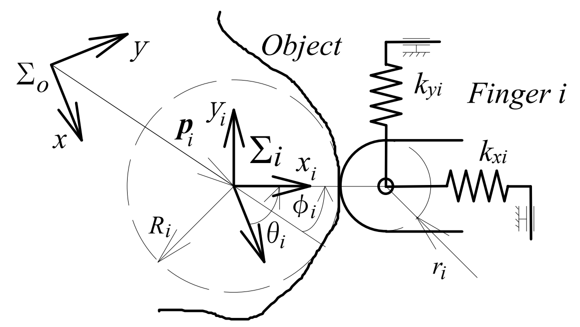


Fig. 5 A grasp with friction point contact

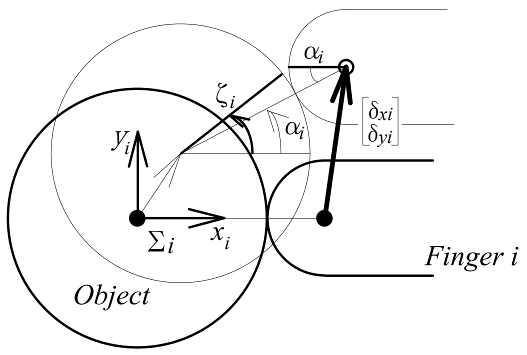


Fig. 6 Rolling contact

3.2. Compression of virtual springs

We will derive the potential energy of the grasp system. So, the compression δ_{xi} and δ_{yi} due to the object displacement will be investigated.

By assumptions (3.1) and (3.2), the positions of the object and the i-th finger move as shown in Fig. 6 when the infinitesimal translation (x_i, y_i) and rotation ζ_i occur. The following equations are obtained from geometric constraints.

$$R_i + r_i + \delta_{xi} = x_i + (R_i + r_i)\cos\alpha_i,$$
$$\delta_{yi} = y_i + (R_i + r_i)\sin\alpha_i,$$
$$r_i\alpha_i = R_i(\zeta_i - \alpha_i) \qquad (15)$$

Since the relation between (x_i, y_i, ζ_i) and (x, y, ζ) is represented by Eqs. (9) and (10), the compression δ_{xi} and δ_{yi} are given by

$$\delta_{xi} = x\cos\theta_i + y\sin\theta_i + p_i\{\cos(\zeta - \phi_i) - \cos\phi_i\} \\ -(R_i + r_i)(1 - \cos\alpha_i),$$
$$\delta_{yi} = -x\sin\theta_i + y\cos\theta_i + p_i\{\sin(\zeta - \phi_i) + \sin\phi_i\} \\ +(R_i + r_i)\sin\alpha_i,$$
$$\alpha_i = \frac{R_i}{R_i + r_i}\zeta \qquad (16)$$

Equation (16) expresses the relation between an object displacement (x, y, ζ) and the compression δ_{xi} and δ_{yi}.

3.3. Evaluation of grasp stability

The potential energy of the grasp system is given by

$$U = \sum U_i$$
$$= \sum\{\tfrac{1}{2}k_{xi}(\delta_{xi} + \delta_{xoi})^2 + \tfrac{1}{2}k_{yi}(\delta_{yi} + \delta_{yoi})^2\} \qquad (17)$$

where δ_{xoi} and δ_{yoi} are initial compression at the equilibrium state. The grasp is in stable configuration if and only if the potential function U is locally minimum at the equilibrium state. By assumption (3.3), $\nabla U|_{(0)} = 0$ is always satisfied. The elements of $H|_{(0)}$ become

$$k_{xx} = \sum(k_{xi}c_i^2 + k_{yi}s_i^2),$$
$$k_{xy} = k_{yx} = \sum(k_{xi} - k_{yi})c_i s_i,$$
$$k_{x\zeta} = k_{\zeta x} = \sum(k_{xi}l_{yi}c_i - k_{yi}l_{xi}s_i),$$
$$k_{yy} = \sum(k_{xi}s_i^2 + k_{yi}c_i^2),$$
$$k_{y\zeta} = k_{\zeta y} = \sum(k_{xi}l_{yi}s_i + k_{yi}l_{xi}c_i),$$
$$k_{\zeta\zeta} = \sum\{k_{xi}l_{yi}^2 + k_{yi}l_{xi}^2 - f_{xi}(l_{xi} - \frac{R_i r_i}{R_i + r_i}) + f_{yi}l_{yi}\}, \qquad (18)$$

where

$$l_{xi} = p_i\cos\phi_i + R_i,\; l_{yi} = p_i\sin\phi_i,$$

$$c_i = \cos\theta_i,\ s_i = \sin\theta_i,$$
$$f_{xi} = k_{xi}\delta_{xoi},\ f_{yi} = k_{yi}\delta_{yoi}$$

$f_{xi} = k_{xi}\delta_{xoi}$ and $f_{yi} = k_{yi}\delta_{yoi}$ are the initial grasping forces along x_i and y_i axis respectively. l_i means the distance between the origin of Σ_o and the i-th contact point.

When the value of k_{xi}, k_{yi}, f_{xi}, f_{yi}, R_i, r_i, $\boldsymbol{p}_i$ are given, we can compute $H|_{(0)}$. If all the eigenvalues of $H|_{(0)}$ are positive, the grasp is in stable configuration.

3.4. Stiffness constants for stabilizing the grasp

As shown in section 3.3, whether the grasp is in stable configuration or not can be evaluated when the stiffness constants k_{xi} and k_{yi} are given. In this section, we will determine the condition of stiffness constants to stabilize the grasp. This analysis is difficult because the elements of $H|_{(0)}$ are complex. For simplicity, we make the following assumptions in addition to assumptions (3.1) -(3.3).

(3.4) The object is held by a three-finger hand. The contact points between the object and the fingers are not aligned.

(3.5) Each x_i axis goes through the same point, and the initial finger force is along x_i.

(3.6) There is no interference between translation and rotation of the object.

By assumption (3.5), we have $\delta_{yoi} = 0$. When the origin of Σ_o is fixed at the intersection of x_i, we have $\sin\phi_i = 0$.

By assumptions (3.4) and (3.5), the elements of $H|_{(0)}$ are simplified as follows:

$$k_{xx} = \sum_{i=1}^{3}(k_{xi}c_i^2 + k_{yi}s_i^2),$$
$$k_{xy} = k_{yx} = \sum_{i=1}^{3}(k_{xi} - k_{yi})c_i s_i,$$
$$k_{x\zeta} = k_{\zeta x} = -\sum_{i=1}^{3} l_{xi}k_{yi}s_i,$$
$$k_{yy} = \sum_{i=1}^{3}(k_{xi}s_i^2 + k_{yi}c_i^2),$$
$$k_{y\zeta} = k_{\zeta y} = \sum_{i=1}^{3} l_{xi}k_{yi}c_i,$$
$$k_{\zeta\zeta} = \sum_{i=1}^{3}\{k_{yi}l_{xi}^2 - f_{xi}(l_{xi} - \frac{R_i r_i}{R_i + r_i})\}, \qquad (19)$$

The matrix $H|_{(0)}$ is positive definite if and only if the following three conditions are satisfied.

(1) $k_{xx} > 0$

(2) $k_{xx}k_{yy} - k_{xy}k_{yx} > 0$

(3) $\det H|_{(0)} > 0$

From Eq. (19), conditions (1) and (2) are always satisfied. However, condition (3) is not always satisfied. This implies that stiffness constants k_{xi} and k_{yi} have lowest limits. Consequently, we investigate the limits.

By assumption (3.6), we have

$$k_{x\zeta} = k_{\zeta x} = -\sum_{i=1}^{3} l_i k_{yi}s_i = 0,$$
$$k_{y\zeta} = k_{\zeta y} = \sum_{i=1}^{3} l_i k_{yi}c_i = 0. \qquad (20)$$

Hence, the ratio of k_{yi} is constrained by the following straight line. Arbitrariness of k_{yi} is represented by magnitude β.

$$\begin{bmatrix} k_{y1} \\ k_{y2} \\ k_{y3} \end{bmatrix} = \beta \begin{bmatrix} \sin(\theta_3 - \theta_2)/l_1 \\ \sin(\theta_1 - \theta_3)/l_2 \\ \sin(\theta_2 - \theta_1)/l_3 \end{bmatrix}. \qquad (21)$$

Then $H|_{(0)}$ becomes

$$H|_{(0)} = \begin{bmatrix} k_{xx} & k_{xy} & 0 \\ k_{yx} & k_{yy} & 0 \\ 0 & 0 & k_{\zeta\zeta} \end{bmatrix}. \qquad (22)$$

Since $k_{xx} > 0$ and $k_{xx}k_{yy} - k_{xy}k_{yx} > 0$ are always satisfied, the condition for satisfying $\det H|_{(0)} > 0$ is given by

$$k_{\zeta\zeta} = \sum_{i=1}^{3}\{k_{yi}l_{xi}^2 - f_{xi}(l_{xi} - \frac{R_i r_i}{R_i + r_i})\} > 0. \qquad (23)$$

From Eqs. (21) and (23), the lowest limit of β for stabilizing the grasp is given by

$$\beta > \frac{\sum_{i=1}^{3} f_{xi}(l_i - \frac{R_i r_i}{R_i + r_i})}{l_1\sin(\theta_3 - \theta_2) + l_2\sin(\theta_1 - \theta_3) + l_3\sin(\theta_2 - \theta_1)} = \beta_o \qquad (24)$$

Therefore, the grasp is always in stable configuration when stiffness constant k_{yi} satisfying Eq. (24) is set. Note that stiffness constant k_{xi} can be set at an arbitrary positive value.

We will compare the proposed method with Refs. [8] and [9]. Since Refs. [8] and [9] assumed that the object is held by sharp fingers, we have $r_i = 0$. Hence, the lowest limit of β is represented by

$$\beta > \frac{\sum_{i=1}^{3} f_{xi} l_i}{l_1 \sin(\theta_3 - \theta_2) + l_2 \sin(\theta_1 - \theta_3) + l_3 \sin(\theta_2 - \theta_1)} = \beta_p \qquad (25)$$

Since we have

$$\beta_p \geq \beta_o, \qquad (26)$$

the curvature of the fingers decrease the lowest limit of k_{yi}. Therefore, our method decreases the energy consumed in the grasp system.

4. Conclusions

We have analyzed the grasp stability by considering the curvature of both hand and object at contact points and the grasp with friction and frictionless point contact. Using the potential energy, the analyses are greatly simplified. From these analyses, it is shown that the grasp using round fingers is more stable than using sharp fingers. Moreover, it is proved that, for a curvature grasps with friction, the required stiffness of fingers are decreased.

This paper assumed no interference between translation and rotation of the grasped object. We will establish the condition of stiffness constants with the interference in the future work. Our current interesting problem is the extension of the planar problem in the paper to the spatial problem.

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Table 1 Numerical Examples

	Forces		Angle & Position		Curvature		Eigenvalues & Stability
i	k_i	δ_{oi}	θ_i	$\boldsymbol{p}_i$	r_i	R_i	
1	1	1.00e-2	$\tan^{-1} 3$	(0.1, 0.3)	-0.4	0.2	(2.0, 1.1, 0.0086)
2	1	3.16e-3	π	(-0.2, 0.0)	0.1	0.2	Stable
3	1	9.49e-3	$3\pi/2$	(0.0, $-0.2\sqrt{3}$)	0.1	-0.2	