Stiffness Condition for Stabilization of Grasps with Interference Between Translational and Rotational Displacements

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Abstract

This paper establishes the general condition of the spring stiffness that makes a spatial grasp stable by allowing interference. Three orthogonal virtual springs, whose stiffness is accomplished by computer control, are fixed at the fingertip. The condition of the stiffness that makes the stiffness matrix positive definite is analyzed. And an assignment procedure of the spring stiffness is established. It is shown that the set of admissible values of the stiffness is greatly extended, and assignment error of the stiffness may be permitted. Therefore, the proposed method needs less energy to grasp. The proposed method is simple and is useful for practical use.

Key Words: grasp stability, potential energy approach, stiffness condition, assignement procedure, less energy.

1. Introduction

While a multifingered robot hand grasps an object, the stability is of essential issue. That is, the hand must not break the contact and not drop the object due to external disturbances. In order to evaluate stability of the grasp, potential energy approach has been proposed by many authors. These methods evaluate whether the grasp will return to the initial state after the disturbances disappeared.

Hanafusa et al. [1] analyzed the stability of frictionless grasp of a multifingered hand with elastic fingers. It was shown that the grasp is stable when the potential energy stored in the grasp is local minimum. Nguyen [2] proposed that stiffness of the real springs is realized by that of virtual springs since the stiffness can be controlled by computers. The stability of the grasp is evaluated by positive definiteness of the stiffness matrix. Kaneko et al. [3] discussed stability of frictional planar grasps and derived stiffness matrix of the grasp. Mimura et al. [4] extended the analysis to 3-D grasps. Funahashi et al. [5] considered the curvature of object and fingertip at the contact points.

Kaneko, Mimura, and Funahashi established spring stiffness that makes the grasp stable, since the stiffness can be assigned any value. In their analyses, however, it is assumed that there is no interference between translational and rotational stiffness. So stiffness of fingertip is restricted.

This paper will establish the stiffness condition of the springs, which makes the spatial grasp stable by allowing interference. Three virtual springs, which are along the normal and the tangential at the contact point, are fixed at the fingertip. First, we derive the stiffness matrix of the frictional grasp. The matrix is influenced by spring stiffness, initial grasping force, and so on. Secondly, the condition of the stiffness that makes the stiffness matrix positive definite is analyzed. Then an assignment procedure of the spring stiffness is proposed. Finally, we will describe the advantage of our method.

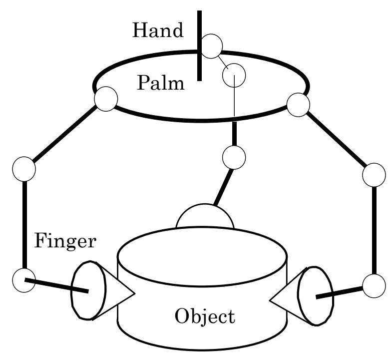

Fig.1 Grasping by a 3-fingered hand

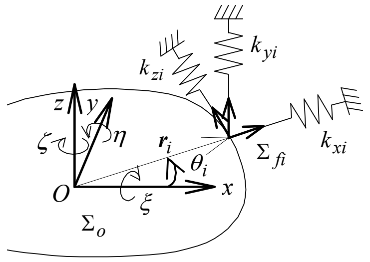

Fig. 2: Orthogonal virtual stiffness at contact point

2. Problem formulation

2.1. Notations

We define the following symbols.

Σ_o : object frame.

Σ_{fi} : i-th finger frame.

R_{fi} : orientation of Σ_{fi} with respect to Σ_o.

$\boldsymbol{f}_i$: contact force.

$\boldsymbol{r}_i$: position of contact point with respect to Σ_o.

θ_i : angle between x axis of Σ_o and $\boldsymbol{r}_i$.

k_{xi}, k_{yi}, k_{zi} : stiffness of springs fixed at i-th fingertip.

$\boldsymbol{\varepsilon} = [\boldsymbol{x}^T \quad \boldsymbol{\xi}^T]^T = [x \quad y \quad z \quad \xi \quad \eta \quad \zeta]^T$:

object displacement due to external distrubances

2.2. Assumptions

An object is grasped by a 3-fingered hand as shown in Fig. 1. Spatial virtual stiffness k_{xi}, k_{yi}, k_{zi} is fixed at the fingertip as shown in Fig. 2. This paper discusses the condition of the virtual springs k_{xi}, k_{yi}, k_{zi} which make the grasp stable. For simplicity of discussions, we make the following assumptions.

(A1) Contact between fingertips and the object is of point contact type with friction.

(A2) Contact position $\boldsymbol{r}_i$ and force $\boldsymbol{f}_i$ are known.

(A3) The grasp system is in equilibrium at the initial configuration.

(A4) 3-points of contact are not aligned.

(A5) The stiffness k_{xi} is assigned to be along the vector $\boldsymbol{r}_i$, k_{yi} lies in the grasp plane, k_{zi} perpendicular to the grasp plane. k_{xi}, k_{yi} and k_{zi} are positive.

From Assumption (A3), initial grasping forces are internal forces. Σ_o is fixed at which $\boldsymbol{f}_i$'s intersect. The x-y plane of Σ_o lies in the grasp plane. An i-th fingertip coordinate Σ_{fi} is fixed along the springs k_{xi}, k_{yi} and k_{zi}.

2.3. Stability of the grasp

Compression of the springs is given by

$$[\delta_{xi} \quad \delta_{yi} \quad \delta_{zi}]^T = R_{fi}^T [\boldsymbol{x} + \{R(\boldsymbol{\xi}) - I_3\}\boldsymbol{r}_i], \tag{1}$$

where

$$R(\boldsymbol{\xi}) = R_z(\zeta) R_y(\eta) R_x(\xi), \quad R_{fi} = R_z(\theta_i),$$

$$\boldsymbol{r}_i = r_i [\cos\theta_i \quad \sin\theta_i \quad 0]^T,$$

$$\boldsymbol{f}_i = -f_i [\cos\theta_i \quad \sin\theta_i \quad 0]^T, \tag{2}$$

The potential energy stored in the grasp system is given by

$$U = \frac{1}{2} \sum_{i=1}^{n} \{k_{xi}(\delta_{xi} + \delta_{xoi})^2 + k_{yi}\delta_{yi}^2 + k_{zi}\delta_{zi}^2\}. \tag{3}$$

Using Taylor series of Eq. (2) around $\varepsilon = 0$, we obtain

$$U = U(0) + \varepsilon^T \nabla U\big|_{(0)} + \tfrac{1}{2}\varepsilon^T H\big|_{(0)} \varepsilon + \cdots, \tag{4}$$

where

$$\nabla = \frac{\partial}{\partial \boldsymbol{\varepsilon}} \in \Re^6, \quad H = \nabla \nabla^T U \in \Re^{6\times 6}. \tag{5}$$

The grasp is stable if and only if the energy U is local minimum at the initial condition $(\varepsilon = 0)$, that is, the following two conditions are satisfied.

(i) $\nabla U\big|_{(0)} = 0$.

(ii) $H\big|_{(0)}$ is positive definite.

From Assumption (A2), the condition (i) is always satisfied. Consequently, the grasp is stable if the condition (ii) is satisfied. The Hessian $H\big|_{(0)}$ is given by

$$H\big|_{(0)} = \begin{bmatrix} k_{xx} & k_{xy} & 0 & 0 & 0 & k_{x\zeta} \\ k_{yx} & k_{yy} & 0 & 0 & 0 & k_{y\zeta} \\ 0 & 0 & k_{zz} & k_{z\xi} & k_{z\eta} & 0 \\ 0 & 0 & k_{\xi z} & k_{\xi\xi} & k_{\xi\eta} & 0 \\ 0 & 0 & k_{\eta z} & k_{\eta\xi} & k_{\eta\eta} & 0 \\ k_{\zeta x} & k_{\zeta y} & 0 & 0 & 0 & k_{\zeta\zeta} \end{bmatrix},$$

where

$k_{xx} = \sum k_{xi} c_i^2 + k_{yi} s_i^2$, $k_{xy} = k_{yx} = \sum (k_{xi} + k_{yi}) c_i s_i$,

$k_{x\zeta} = k_{\zeta x} = -\sum k_{yi} r_i s_i$, $k_{yy} = \sum k_{xi} s_i^2 + k_{yi} c_i^2$

$k_{y\zeta} = k_{\zeta y} = \sum k_{yi} r_i c_i$, $k_{zz} = \sum k_{zi}$,

$k_{z\xi} = k_{\xi z} = \sum k_{zi} r_i s_i$, $k_{z\eta} = k_{\eta z} = -\sum k_{zi} r_i c_i$,

$k_{\xi\xi} = \sum(k_{zi}r_i - f_i)r_i s_i^2$, $k_{\xi\eta} = \sum(-k_{zi}r_i + f_i)r_i c_i s_i$,
$k_{\eta\eta} = \sum(k_{zi}r_i - f_i)r_i c_i^2$, $k_{\zeta\zeta} = \sum k_{yi}r_i^2 - f_i r_i$,
where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$.

The elements of $H\big|_{(0)}$ depend on the stiffness of k_{xi} , k_{yi} , and k_{zi} . In the following sections, we will derive the condition of stiffness k_{xi} , k_{yi} , and k_{zi} , which makes the matrices $H\big|_{(0)}$ positive definite.

3. Stiffness Condition

Exchanging rows and columns of $H\big|_{(0)}$ yields

$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix},$$

where

$$K_1 := \begin{bmatrix} k_{xx} & k_{xy} & k_{x\zeta} \\ k_{yx} & k_{yy} & k_{y\zeta} \\ k_{\zeta x} & k_{\zeta y} & k_{\zeta\zeta} \end{bmatrix}, \quad K_2 := \begin{bmatrix} k_{zz} & k_{z\xi} & k_{z\eta} \\ k_{\xi z} & k_{\xi\xi} & k_{\xi\eta} \\ k_{\eta z} & k_{\eta\xi} & k_{\eta\eta} \end{bmatrix}$$

The following two conditions are equivalent.

(i) $H\big|_{(0)}$ is positive definite.

(ii) K_1 and K_2 are positive definite.

We will analyze the matrices K_1 and K_2 .

3.1. Condition of k_{xi} and k_{yi}

The matrix K_1 can be rewritten as

$$\begin{aligned} K_1 &= \begin{bmatrix} -s_1 & -s_2 & -s_3 \\ c_1 & c_2 & c_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{bmatrix} k_{y1} & 0 & 0 \\ 0 & k_{y2} & 0 \\ 0 & 0 & k_{y3} \end{bmatrix} \begin{bmatrix} -s_1 & c_1 & r_1 \\ -s_2 & c_2 & r_2 \\ -s_3 & c_3 & r_3 \end{bmatrix} \\ &+ \begin{bmatrix} c_1 & c_2 & c_3 \\ s_1 & s_2 & s_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_{x1} & 0 & 0 \\ 0 & k_{x2} & 0 \\ 0 & 0 & k_{x3} \end{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ c_2 & s_2 & 0 \\ c_3 & s_3 & 0 \end{bmatrix} \\ &- \sum_{i=1}^{3} r_i f_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \\ &= A K_y A^T + B K_x B^T - d\boldsymbol{c}\boldsymbol{c}^T \qquad (6) \end{aligned}$$

where

$K_x = diag[k_{x1}, k_{x2}, k_{x3}]$, $K_y = diag[k_{y1}, k_{y2}, k_{y3}]$,

$$A = \begin{bmatrix} -s_1 & -s_2 & -s_3 \\ c_1 & c_2 & c_3 \\ r_1 & r_2 & r_3 \end{bmatrix}, \quad B = \begin{bmatrix} c_1 & c_2 & c_3 \\ s_1 & s_2 & s_3 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{c} = [0 \quad 0 \quad 1]^T, \quad d = \sum_{i=1}^{3} r_i f_i .$$

Since the matrix A is nonsingular from Assumption (A6), we define P as

$$\begin{aligned} P(k_{xi}, k_{yi}) &:= A^{-1} K_1 A^{-T} \\ &= K_y + \tilde{B} K_x \tilde{B}^T - d\tilde{\boldsymbol{c}}\tilde{\boldsymbol{c}}^T , \qquad (7) \end{aligned}$$

where $\tilde{B} := A^{-1}B$ and $\tilde{\boldsymbol{c}} := A^{-1}\boldsymbol{c}$. Then we have the following condition.

$$K_1 > 0 \quad \Leftrightarrow \quad P > 0$$

Therefore, we derive the condition of k_{xi} and k_{yi} which makes $P(k_{xi}, k_{yi})$ positive definite. From

$$P > 0 \quad \Leftrightarrow \quad \boldsymbol{l}^T P \boldsymbol{l} > 0 \quad \text{for} \quad \boldsymbol{l} = [l_1 \ l_2 \ l_3]^T \neq 0 ,$$

we investigate $\boldsymbol{l}^T P \boldsymbol{l}$. Then we obtain

$$\begin{aligned} \boldsymbol{l}^T P \boldsymbol{l} &= k_{y1} l_1^2 + k_{y2} l_2^2 + k_{y3} l_3^2 \\ &+ k_{x1}\{(\tilde{B}^T \boldsymbol{l})_1\}^2 + k_{x2}\{(\tilde{B}^T \boldsymbol{l})_2\}^2 + k_{x3}\{(\tilde{B}^T \boldsymbol{l})_3\}^2 \\ &- d(\boldsymbol{l}^T \tilde{\boldsymbol{c}})^2 \qquad (8) \end{aligned}$$

where $(\tilde{B}^T \boldsymbol{l})_i$ denotes the i-th element of the vector $\tilde{B}^T \boldsymbol{l}$. Since, from Eq. (8), $\boldsymbol{l}^T P \boldsymbol{l}$ is linear with respect to k_{xi} and k_{yi} and is non-negative, there exists lowest limit of k_{xi} and k_{yi} . The coefficient of k_{xi} will be zero for $\boldsymbol{l} \in Ker\tilde{B}^T$. The matrix $P(k_{xi}, k_{yi})$ is not guaranteed to be positive definite by enlarging k_{xi} . However, k_{yi} is guaranteed. Consequently, the grasp is stable if stiffness of the springs is assigned as follows. First, k_{xi} 's are assigned any values, then we have

$$Q = \{q_{ij}\} := -\tilde{B} K_x \tilde{B}^T + d\tilde{\boldsymbol{c}}\tilde{\boldsymbol{c}}^T \qquad (9)$$

k_{y1} is assigned to be

$$k_{y1} > q_{11} \qquad (10)$$

Then k_{y2} is assigned to be

$$k_{y2} > \frac{q_{12}^2}{k_{y1} - q_{11}} + q_{22} \qquad (11)$$

Finally, k_{y3} is assigned to be

$$k_{y3} > \frac{q_{23}^2 (k_{y1} - q_{11}) + q_{13}^2 (k_{y2} - q_{22}) + 2 q_{12} q_{13} q_{23}}{(k_{y1} - q_{11})(k_{y2} - q_{22}) - q_{12}^2} + q_{33} \qquad (12)$$

We pick up two of k_{y1} , k_{y2} , and k_{y3} , these are in inverse proportion to each other. Therefore, possible region of k_{yi} is given by upper area of the bound shown in Fig. 3.

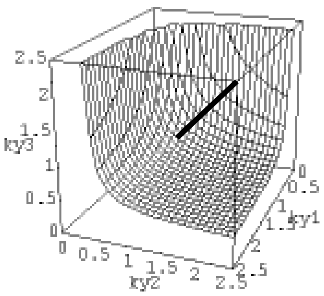

Fig. 3: possible set area of k_{yi}

Refs. [3]-[5] assumed

$$k_{x\zeta} = k_{\zeta x} = 0, \quad k_{y\zeta} = k_{\zeta y} = 0 .$$

These mean no interference between translational and rotational stiffness. So the stiffness k_{y1}, k_{y2}, and k_{y3} are limited to the segment illustrated in Fig. 3. However, our analysis shows that the grasp can be originally stabilized in the wide area of stiffness. This means that we can assign any values of stiffness from the set satisfying Eqs. (10) - (12) and assignment error of stiffness may be permitted when a multifingered robot hand is controlled.

3.2. Condition of k_{zi}

The matrix K_2 can be represented by

$$K_2 = \begin{bmatrix} 1 & 1 & 1 \\ r_1 s_1 & r_2 s_2 & r_3 s_3 \\ -r_1 c_1 & -r_2 c_2 & -r_3 c_3 \end{bmatrix} \begin{bmatrix} k_{z1} & 0 & 0 \\ 0 & k_{z2} & 0 \\ 0 & 0 & k_{z3} \end{bmatrix} \begin{bmatrix} 1 & r_1 s_1 & -r_1 c_1 \\ 1 & r_2 s_2 & -r_2 c_2 \\ 1 & r_3 s_3 & -r_3 c_3 \end{bmatrix}$$

$$- \sum_{i=1}^{3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & f_i r_i s_i^2 & -f_i r_i c_i s_i \\ 0 & -f_i r_i c_i s_i & f_i r_i c_i^2 \end{bmatrix}$$

$$= A' K_z A'^T - C . \qquad (13)$$

Since, from Assumption (A4), the matrix A is nonsingular, we have the following condition.

$$K_2 > 0 \iff K_z - A'^{-1} C A'^{-T} > 0 .$$

The grasp can be stabilized, if k_{zi} are assigned to be

$$k_{z1} > c'_{11}, \quad k_{z2} > \frac{{c'_{12}}^2}{k_{z1} - c'_{11}} + c'_{22},$$

$$k_{z3} > \frac{{c'_{23}}^2 (k_{z1} - c'_{11}) + {c'_{13}}^2 (k_{z2} - c'_{22}) + 2 c'_{12} c'_{13} c'_{23}}{(k_{z1} - c'_{11})(k_{z2} - c'_{22}) - {c'_{12}}^2} + c'_{33},$$

where $C' = \{c'_{ij}\} := A'^{-1} C' A'^{-T}$.

Therefore all the values of stiffness are determined. Due to lack of the space, numerical examples are not described. From the examples, however, it is ensured that our method can stabilize the grasp more easily with less energy than the methods of Refs. [3]-[5].

4. Conclusion

We have investigated stabilization of the 3D grasp of 3 fingered with friction contact. Three orthogonal virtual stiffness are fixed to stabilize the grasp. We obtained the general solution for stabilization of grasp. The main results of this paper are as follows.

(1) We allow the interference between translational and rotational stiffness. Hence, admissible region of stiffness is greatly extended.

(2) We can assign the values of stiffness from the wide area. The assignment error may be permitted. Our method needs less energy to grasp.

Since the stiffness of the fingertip is accomplished by computer control, the stabilization method proposed in this paper is simple and is useful for the grasp by a multifingered hand.

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