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Bayes Decision Procedure Model for Post-Earthquake Emergency Response

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Abstract

A Bayes decision procedure model is proposed for optimizing the process of post-earthquake emergency response in highly uncertain conditions. Emergency shut-off of lifeline services is an important measure to prevent secondary damage. However, the decision maker encounters the risk resulting from overestimate and underestimate of the damage. Assuming that the decision maker can acquire information on seismic intensity, the Bayes decision procedure is formulated as the most appropriate strategy of actions to minimize the expected loss. The information theory has been employed to quantify the degree of uncertainty and the value of information. Numerical examples suggest that (1) the proposed model serves as a useful decision support system, (2) prompt acquisition of information is essential to emergency response, (3) repeated acquisition of information reduces uncertainty measured in terms of entropy, and (4) the marginal value of information gradually decreases.

Introduction

Post-earthquake emergency response is an important process in order to prevent occurrence of secondary disaster and in order to mitigate physical/functional and time/spatial spread of damage. Emergency shut-off of service is one of those typical actions taken by lifeline service providers such as city gas supply, water supply, highway transportation, and railway transportation in the case of severe earthquake (Nojima and Kameda, 1997). Because such a decision is inevitably made on the basis of uncertain information on the actual states of damage, emergency responders may be involved in a dilemma; (1) overestimate of damage leads to unnecessary shut-off of service, and (2) underestimate of damage leads to fatal default of prepared actions. In general, damage information which is available immediately after the earthquake is

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deficient, rough, biased and uncertain. Decision makers gradually obtain abundant, detailed, unbiased, and certain information using various sources of information. In the final stage, perfect knowledge on the actual damage state is achieved, but mostly it is too late to take an action. Then, a question arises; how should emergency responders make a decision in highly uncertain conditions ? And when ?

To answer this question, this study introduces a “Bayes decision procedure model” which provides an engineering rationale to a post-earthquake emergency response strategy based on the statistical decision theory and the information theory. First, a theoretical framework of Bayes decision procedure is described. A Bayes decision procedure gives “the optimized rule of observations and actions,” in a sense that the expected loss can be minimized for an arbitrary combination of observation and action. Next, the concept of entropy is introduced as a quantitative measure of uncertainty. Since there exists a conflict between promptness and accuracy in a decision making process, assessment of uncertainty is essential consideration for engineering judgment of post-earthquake emergency actions. In this study, a process of accumulation of information, which is equivalent to reduction of uncertainty, is represented by a decreasing process of entropy. The value of information is also quantified in terms of the degree of reduction of expected loss due to the information. Numerical examples are shown for illustrative demonstration.

Model of Post-Earthquake Emergency Response

Suppose that an earthquake has occurred, and an operator of lifeline facility is facing a problem whether or not to shut off services. **Figure 1** illustrates the relationship among the factors considered herein; seismic intensity, the information, damage levels, actions, and losses. When the actual damage level (denoted by q) is known, the decision maker immediately takes an action (denoted by a) of shut-off (or partially shut-off / continue) service to minimize the loss caused by the combination of damage and the action (denoted by $\ell(q, a)$). However, when the actual damage level q is unknown, or at least uncertain, the decision maker tries to estimate damage using

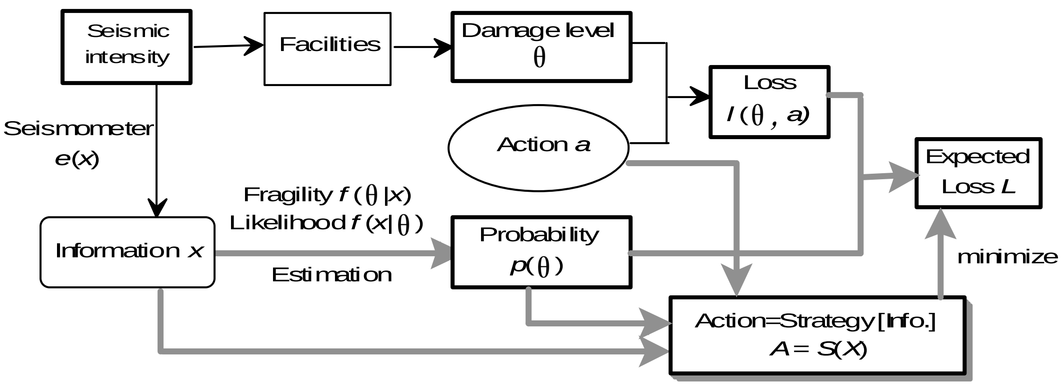


Figure 1. Estimation of Damage and Actions to Minimize the Expected Loss

seismic intensity data (denoted by x) obtained from seismometers (denoted by $e(x)$). As a result of series of information acquisition, the decision maker guesses that the damage is severe (or slight / none) with a subjective probability (denoted by $p(q)$), and then makes a decision to take an action (denoted by a) in order to minimize the expected loss (denoted by L). In this context, the subjective probability $p(q)$ and the seismic intensity data x prescribes a decision rule (denoted by $A = S(X)$). During the whole process, the decision maker integrates the sequence of information to update one's subjective probability of the damage level. The final decision depends on a trade-off between accuracy of estimation and promptness of action.

This scenario can be appropriately modeled as a decision problem (A, Θ, p, ℓ, e) defined by the following components (Chernoff and Moses, 1959):

- a set of possible states of nature [damage levels] q ($\Theta = \{q_1, q_2, \cdots\}$),
- a set of possible observation data [seismic intensity levels] x ($X = \{x_1, x_2, \cdots\}$),
- a set of feasible actions [shut-off levels] a ($A = \{a_1, a_2, \cdots\}$),
- subjective probability of Θ and X ($p(q) = \{p(q_1), p(q_2), \cdots\}$, $p(x) = \{p(x_1), p(x_2), \cdots\}$),
- decision rule [action determined as a function of observation data x] $A = S(X)$,
- loss function [loss caused by taking action a under the actual damage state q] $\ell(q, a)$,
- information source [seismometers] $e(x)$,
- likelihood function [probability of obtaining observation x under the state q] $f(x|q)$.

Formulation of a Bayes Decision Procedure

Figure 2 illustrates the sequential process of updating subjective probability $p(q_j)$ on the basis of the obtained information and decision making of action according to the the Bayes decision procedure (Chernoff and Moses, 1959) described below. If there is no information available, the most rational action a^* is such that minimizes the

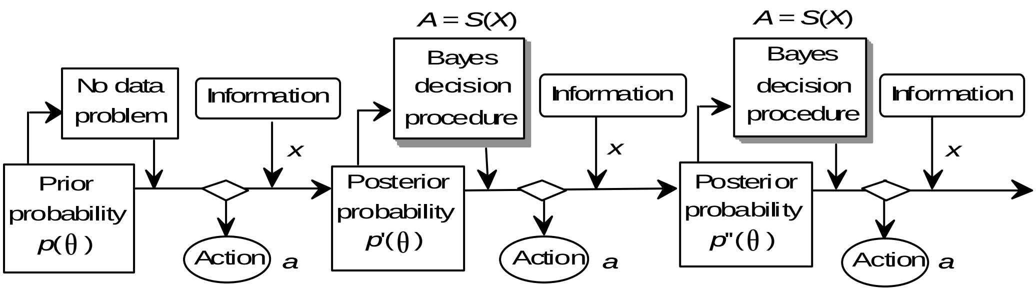


Figure 2. The Process of Updating Probability Using Information and Bayes' Theorem

expected loss L which is evaluated by averaging the loss function $\ell(q_j, a)$ over the prior probability $p(q_j)$ of each state q_j.

$$L = \sum_j p(q_j)\,\ell(q_j, a^*) \rightarrow \text{minimize}$$

This simple case is referred to as a "no data problem." On the other hand, when an information source $e(x)$ is available for estimating the damage state, the expected loss can be written as the following equation.

$$L = \sum_i \sum_j f(x_i \mid q_j)\,p(q_j)\,\ell(q_j, S(x_i))$$

When an observation data x_i is obtained, the prior probability $p(q)$ is updated to the posterior probability $p(q_j \mid x_i)$ according to the Bayes' theorem (Ang and Tang, 1975):

$$p(q_j \mid x_i) = \frac{f(x_i \mid q_j)\,p(q_j)}{\sum_j f(x_i \mid q_j)\,p(q_j)} = \frac{f(x_i \mid q_j)\,p(q_j)}{p(x_i)}$$

Hence, the expected loss can be rewritten as:

$$L = \sum_i \sum_j p(q_j \mid x_i)\,p(x_i)\,\ell(q_j, S(x_i)) = \sum_i p(x_i) \sum_j p(q_j \mid x_i)\,\ell(q_j, S(x_i)) = \sum_i p(x_i) L_i$$

By performing termwise minimization according to the observation x_i, i.e.,

$$L_i^* = \sum_j p(q_j \mid x_i)\,\ell(q_j, a_x) \rightarrow \text{minimize}$$

the Bayes decision procedure $a_x = S^*(x_i)$ can be obtained on the basis of the posterior probability $p(q_j \mid x_i)$.

Quantification of Uncertainty and the Value of Information

Benefit of obtaining the observation data x is interpreted that such information can reduce the degree of uncertainty in estimating the actual state q. In the information theory, the degree of uncertainty is quantified using "entropy" (Shannon, 1964) Entropy for the probability distribution $p(q) = \{p(q_1), p(q_2), \cdots\}$ is defined as the following equation.

$$H[p(q)] = -\sum_j p(q_j) \log_2 p(q_j)$$

The amount of information of observation x using the information source $e(x)$ is represented by "mutual information" defined as the difference between unconditional entropy and entropy conditioned by x.

$$\begin{aligned} I[\Theta; X] &= H[p(q)] - H[p(q) \mid x] \\ &= -\sum_j p(q_j) \log_2 p(q_j) + \sum_i \sum_j p(x_i, q_j) \log_2 p(q_j \mid x_i) \end{aligned}$$

The value of information (Ang and Tang, 1984) is quantified as the difference between expected losses with and without the information. Let $e_\infty(x)$ denote an information source providing the perfect information on the actual damage state q. The optimum action that minimizes the expected loss where the actual damage state q is known using $e_\infty(x)$ is denoted by a_q. The optimum actions that minimize the expected losses in cases where $e(x)$ are available and unavailable are denoted by a_x and a^*, respectively. With these notations, the vaule of information of $e_\infty(x)$ and $e(x)$ are defined by

$$V(e_\infty(x)) = \mathrm{E}[\ell(q, a_q), p(q)] - \mathrm{E}[\ell(q, a^*), p(q)]$$
$$V(e(x)) = \mathrm{E}[\ell(q, a_x), p(x)] - \mathrm{E}[\ell(q, a^*), p(q)]$$

where $\mathrm{E}[\ell, p(\bullet)]$ represents expected value of ℓ averaged over a probability distribution $p(\bullet)$.

Hypothetical Decision Problem for Numerical Examples

Assume a hypocetical decision problem of shut-off action with an available information of seismic intensity on JMA (Japan Meteorological Agency) scale. Damage levels q are categorized into three: "(1) severe," "(2) minor," and "(3) none." The observation data of seismic intensity scale x for damage estimation is categorized into four: "(1) V weak", "(2) V strong", "(3) VI weak", and "(4) VI strong." Feasible shut-off actions a are three-fold: "(1) complete shut-off," "(2) partial shut-off," and "(3) no shut-off."

The likelihood function $f(x|q)$ is shown in **Table 1(a),(b)** . **Table 1(a)** is an example of the perfect information. Because non-zero values appear only once in each column, the damate state is perfectly identified according to the information of seismic intensity. On the other hand, **Table 1(b)** shows an ordinary information source in which uncertainty is involved to some extent. The values distribute over multiple levels of seismic intensity due to uncertainty. Note that the sum of each row equals unity in both cases.

Table 1. Likelihood Functions of Information

(a) Perfect information

Damage	V-	V+	VI-	VI+
Severe	0.0	0.0	0.0	1.0
Minor	0.0	0.5	0.5	0.0
None	1.0	0.0	0.0	0.0

(b) Uncertain information

Damage	V-	V+	VI-	VI+
Severe	0.0	0.0	0.2	0.8
Minor	0.0	0.1	0.5	0.4
None	0.4	0.3	0.2	0.1

In this study, the loss function $\ell(q, a)$, shown in **Table 2(a)**, is decomposed into three different terms and is described as a time-dependent function:

$$\ell(q, a, t) = \ell_0(q) + \ell_a(q, a) + \ell_b(q, a_0) \cdot g(t) + \ell_b(q, a) \cdot \{1 - g(t)\}$$

In this equation, the term $\ell_0(q)$ represents the initial loss due to physical damage to the facilities. As **Table 2(b)** shows, the initial loss is considered to be independent of

Table 2. Loss Functions and Three Decomposed Elements

(a) Total loss

Damage	Complete Shut-off	Partial Shut-off	No Shut-off
Severe	40	60	100
Minor	30	10	30
None	20	10	0

(b) Initial loss

Damage	Complete Shut-off	Partial Shut-off	No Shut-off
Severe	40	40	40
Minor	10	10	10
None	0	0	0

(c) Loss due to unnecessary shut-off

Damage	Complete Shut-off	Partial Shut-off	No Shut-off
Severe	0	0	0
Minor	20	0	0
None	20	10	0

(d) Loss due to default of necessary shut-of

Damage	Complete Shut-off	Partial Shut-off	No Shut-off
Severe	0	20	60
Minor	0	0	20
None	0	0	0

both the action a and time t; it is determined solely on the basis of the damage state q. The second term $\ell_a(q,a)$ is relevant to so-called "producer's risk" (Ang and Tang, 1975) which means additional loss due to the unnecessary shut-off action. As shown in **Table 2(c)**, the loss is generated when the damage is overestimated. On the other hand, the third term $\ell_b(q,a)$ is relevant to so-called "consumer's risk" (Ang and Tang, 1975) which means additional loss due to default of necessary shut-off action. As shown in **Table 2(d)**, the loss is generated when the damage is underestimated. The time function $g(t)$ is introduced in order to model the damage spread with time; a delayed response exercerbates damage. The fourth term $\ell_b(q,a_0)\cdot g(t)$, in which a_0 means the action "no shut-off," represents the accumulated loss caused by suspending decision and leave the damage spreading. Finally, the last term $\ell_b(q,a)\cdot\{1-g(t)\}$ represents the residual loss generated after the action a is taken. A simple form of time function ranging from 0 to 1 is assumed in this study:

$$g(t)=\left(\frac{t}{t_0}\right)^{\left(1-\frac{t}{t_0}\right)q}$$

where t_0 denotes the duration of the damage spread and q denotes the shape parameter. **Figure 3** shows $g(t)$ for various values of q. In the extreme case, when $q=0$, the loss instantaneously reaches to its maximum value, namely, $g(t)\equiv 1$. When $q=\infty$, no penalty for suspending decision making is imposed, namely, $g(t)\equiv 0$. When $q=2$, the loss is almost proportional to the time.

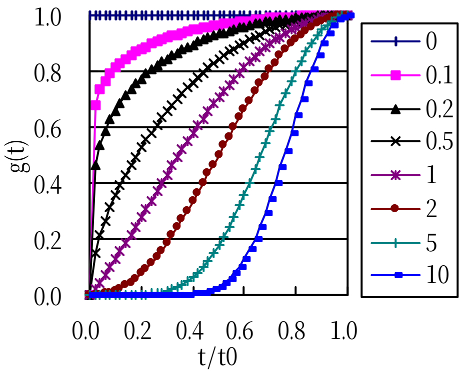


Figure 3. Time Function Describing the Damage Spread

Results

In this section, results are shown for six cases with different values of q, different types of information source, and different sequences of information.

Case 1 : $q = \infty$, perfect information is available

Case 2 : $q = \infty$, no information is available ("no data problem")

Case 3 : $q = \infty$, information sequence = {VI-, VI-, VI-, VI-, VI-}

Case 4 : $q = \infty$, information sequence = {VI+, VI+, VI+, VI+, VI+}

Case 5 : $q = 2$, information sequence = {VI-, VI-, VI-, VI-, VI-}

Case 6 : $q = 2$, information sequence = {VI+, VI+, VI+, VI+, VI+}

1) Case 1

Let us begin with the case where the perfect information source $e_\infty(x)$ whose likelihood function is shown in **Table 1(a)** is available. The damage state is perfectly known according to the information, therefore, the optimum action a_q is definitely determined so that the loss is minimized according to the damage state (**Table 2(a)**):

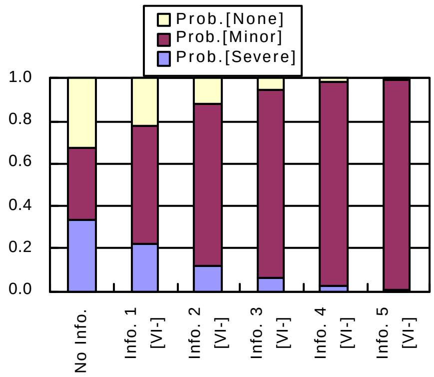
VI+	$\Rightarrow$ severe damage	$\Rightarrow$ complete shut-off	(minimum of {<u>40</u>, 60, 100})
V+, VI-	$\Rightarrow$ minor damage	$\Rightarrow$ partial shut-off	(minimum of {30, <u>10</u>, 30})
V-	$\Rightarrow$ no damage	$\Rightarrow$ no shut-off	(minimum of {20, 10, <u>0</u>})

2) Case 2

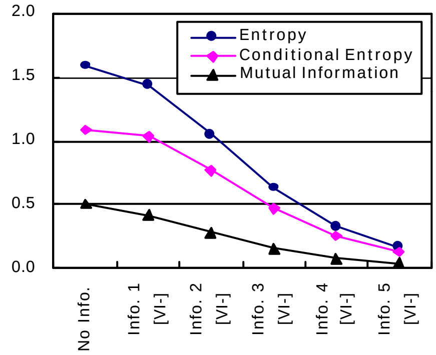
Next, consider that the actual damage state is completely unknown. According to "the principle of complete ignorance," the prior probability (diffuse prior) is given as a uniform distribution $p(q_j) = 1/3 (j = 1,2,3)$. When no data is available, the expected loss resulting from the actions (1), (2) and (3) are evaluated as 30.0, 26.7, and 43.3, respectively (the column averages of **Table 2(a)**). Consequently, the optimum action at the initial stage without any information is decided as "partial shut-off."

3) Case 3

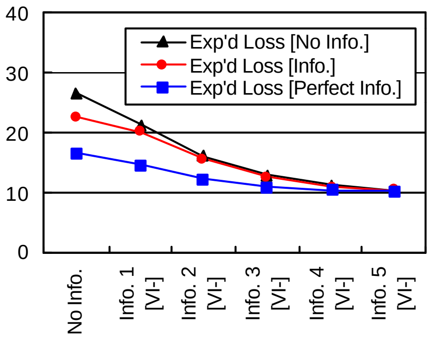
Suppose that an information source $e(x)$ whose likelihood function is shown in **Table 1(b)** is made available so that seismic intensity scale x can be observed. It is assumed that observations sequentially obtained from the information source are mutually independent. Given that all of five observations of seismic intensity are "VI weak," **Figure 4(a)-(d)** shows the overall results. It can be recognized that, as a result of repeated acquisition of the observation data "VI weak," the decision maker becomes confident that "minor damage" occurres (**Figure 4(a)**). In other words, degree of uncertainty related to the estimation of actual damage is reduced with accumulation of information, which is clealy observed as a decreasing process of the vaule of entropy shown in **Figure 4(b)**. **Table 3** shows the Bayes decision procedure $A = S(X)$ which prescribes the most preferable action corresponding to an arbitrary observation. As a result of the sequential acquisition of information, the action "partial shut-off" becomes dominant. As shown in **Figure 4(c)**, the expected loss converges to 10 which corresponds to the loss resulting from the conbination of "minor



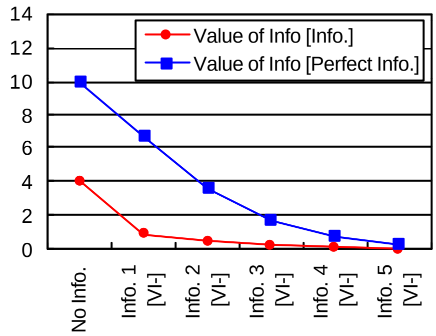
(a) Subjective probability of damage level



(b) Entropy and mutual information



(c) Expected loss



(d) Value of information

Figure 4. Results for the Case 3

Table 3. Bayes Decision Procedure for the Case 3

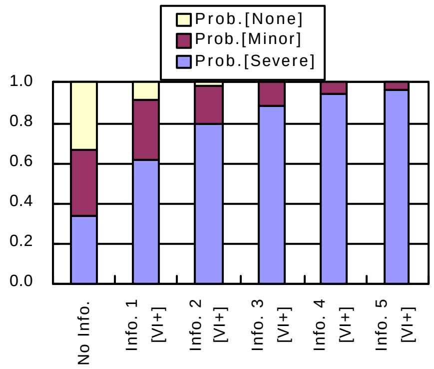
	No Info.	Info. 1 [VI-]	Info. 2 [VI-]	Info. 3 [VI-]	Info. 4 [VI-]	Info. 5 [VI-]
Action [V-]	3	3	3	3	3	3
Action [V+]	3	2	2	2	2	2
Action [VI-]	2	2	2	2	2	2
Action [VI+]	1	2	2	2	2	2

Note: (1) Complete shut-off, (2) Partial shut-off, (3) No shut-off

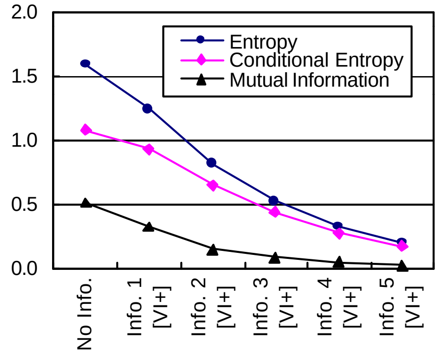
damage" and "partial shut-off" in **Table 2(a)**. At early stages when information is insufficient, both of mutual information and the value of information take large values. As the decision maker acquires sufficient information, however, both values gradually decrease and approach to zero (**Figure 4(b),(d)**).

4) Case 4

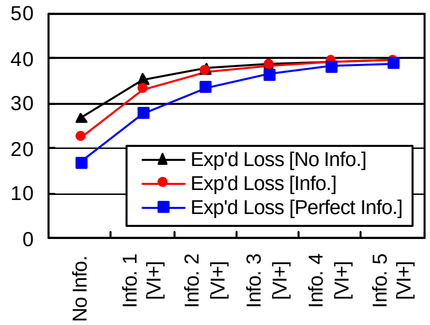
Consider the same condition as the case 3 except that all of five observations of seismic intensity are "VI strong." **Figure 5(a)-(d)** show the overall results. As shown in **Figure 5(a)**, the decision maker gradually becomes sure that "severe damage" occurres. The Bayes decision procedure shown in **Table 4** shifts to a safty-oriented action pattern, namely, "complete shut-off." **Figure 5(c)** shows that the expected loss converges to 40 which corresponds to the loss resulting from the combination of "severe damage" and "complete shut-off" in **Table 2(a)**.



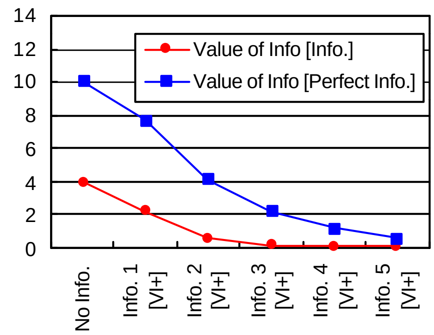
(a) Subjective probability of damage level



(b) Entropy and mutual information



(c) Expected loss



(d) Value of information

Figure 5. Results for the Case 4

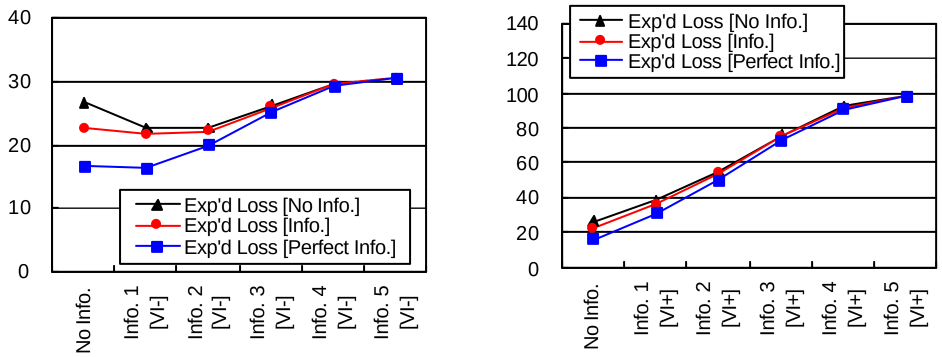
Table 4. Bayes Decision Procedure for the Case 4

	No Info.	Info. 1 [VI+]	Info. 2 [VI+]	Info. 3 [VI+]	Info. 4 [VI+]	Info. 5 [VI+]
Action [V-]	3	3	3	3	3	3
Action [V+]	3	2	2	2	2	2
Action [VI-]	2	2	1	1	1	1
Action [VI+]	1	1	1	1	1	1

Note: (1) Complete shut-off, (2) Partial shut-off, (3) No shut-off

5) Case 5 and Case 6

Consider that the delayed effect of the loss is described by the parameter $q = 2$. These cases simulate the situation that the decision maker recognizes the growing loss in a real-time manner while standing by for taking an action. For simplicity, the duration of damage spread t_0 is normalized to 1, and it is assumed that informations are obtained at $t = 0.2, 0.4, 0.6, 0.8, 1.0$. In the case 5, when "VI weak" is sequentially observed, the expected loss decreases once and then increases up to 30 as shown in **Figure 6(a)**. A hasty action without using information may lead to the "producer's risk." **Figure 6(b)** shows the result of the case 6, when "VI strong" is sequentially observed. The expected loss increases up to 100 which is much larger than the corresponding case shown in **Figure 5(c)**. This result suggests that the



(a) Case 5 (b) Case 6

Figure 6. Expected Loss for the Case 5 and the Case 6

decision must be made as soon as possible in a severe earthquake. In any case, prompt acquisition of information is essential to realize the optimization of an emergency response. Little difference was seen beween the case 3 / 5 and the case 4 / 6 in the results of entropy, mutual information, and the value of information.

Concluding Remarks

This study presented a Bayes decision procedure model for a theoretical interpretation of the information process in post-earthquake emergency response. A decision rule for an emergency action that minimizes the expected loss is determined according to the observed data sequentially updating the subjective probability of damage state (Inconsistent information is of course allowable). The final decision for activating the decision rule can be made with examining the degree of uncertainty, mutual information, and the value of information which are made quantified by the proposed model. In the future, it is expected that dense, various, and precise information on seismic intensity is available, in addition to the conventional information of JMA seismic intensity. The Bayes decision procedure model will serve as a powerful means to appropriately utilize such information for a real-time emergency response aiming at reducing indirect damage spread. In the further study, the proposed model should be improved so as to be applied to more practical situations; a decision problem of shut-off for wide-spread facilities with a variety of information sources of spatial distributions of seismic intensity and damage itself.

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Keywords

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Entropy
Value of information
Expected loss