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Prioritization in Upgrading Seismic Performance of Road Network Based on System Reliability Analysis

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ABSTRACT

Prioritized allocation of limited resources is essential consideration in seismic improvement of existing transportation facilities. In this study, the concept of performance-based prioritization in upgrading seismic reliability of road networks is proposed. Firstly, the performance measure is defined as the system flow capacity of road networks subject to failure. Second, a new variance reduction technique for Monte Carlo simulation method is presented to perform efficient reliability analysis in terms of the system flow capacity. The performance-based prioritization order is then determined by using the ranking of Birnbaum's probabilistic importance measure defined as partial derivative of system reliability by component reliability. Illustrative examples are presented to demonstrate prioritization strategy according to various levels of vulnerability and system requirement.

Keywords : road network, earthquake disaster, system flow capacity, maximum flow, network reliability, importance measure, retrofit prioritization

INTRODUCTION

In the case of earthquake disaster, road traffic function plays a vital role in post-earthquake emergency, recovery, and reconstruction stage. Different from normal situation, traffic capacity is considerably reduced due to both damage to transportation facilities and emergency congestion. Reliable function of road network is prerequisite for success of every kind of post-earthquake effort as well as for normal socioeconomic activities. However, because of time and financial constraints, overall improvement of existing road facilities are impractical. Prioritization in upgrading network components is, therefore, essential consideration for effective implementation of disaster prevention technology.

Basöz and Kiremidjian (1994) presented a method to assess importance of bridges for prioritization in seismic retrofit. Emphasis was placed on the two-step algorithm for (1) finding critical sets of bridges that compose minimal cuts in the transportation network, and (2) ranking individual bridges within the sets. The system functionality was defined as connectivity between critical destinations in cases of emergency. Wakabayashi (1997) carried out importance analysis of Kobe highway network according to several scenarios of link closures. Performance criteria was travel time between Osaka and Kobe, but neither structural vulnerability of network components nor their failure probability was considered.

With this background, the objective of this study is to propose a probabilistic method to assess reliability of road network function in terms of the system flow capacity and to propose a theoretical framework of the concept of performance-based prioritization in upgrading seismic performance of road networks. In the following sections, firstly, maximum flow is introduced as a performance measure of road networks subject to failure.

A binary performance index is also defined on the basis of the maximum flow in order to judge whether the system function satisfies a certain serviceability level or not. Secondly, a hybrid technique which combines the state enumeration method and the Monte Carlo simulation method is proposed to perform high-precision approximation of system reliability. Thirdly, Birnbaum's probabilistic importance measure is introduced to represent probabilistic contribution of upgrading seismic reliability of individual component to that of the system performance. Finally, numerical examples using the proposed method are shown to demonstrate prioritization strategy of upgrading network components with limited resources.

DEFINITION OF ROAD NETWORK PERFORMANCE MEASURE

Flow-dependent and flow-independent performance measures

In conventional traffic engineering, performance of road networks is evaluated through the process of the "four-step estimation method" which includes (1) trip generation and attraction analysis, (2) trip distribution analysis, (3) modal split analysis, and (4) traffic assignment analysis. When detailed and precise information is available, one can estimate network performance in terms of flow-dependent measures such as traffic volume at arbitrary routes or cross sections, and travel time required for arbitrary O/D (origin/destination) pairs. As for the traffic behavior in earthquake disaster, however, it is extremely difficult to predict post-earthquake conditions in detail enough to perform straight-forward application of the "four-step estimation method." Therefore, a simple and easy-to-calculate performance measure is desirable which requires less information of post-earthquake conditions.

Various flow-independent measures have been proposed by Chang and Nojima (1998) to evaluate road network performance. They are (1) total number of highway sections open, (2) total length of highway open, (3) total "connected" length of highway open, and (4) total weighed connected length of highway open. Since these four measures requires only information on pre-earthquake network configuration, pre-earthquake traffic volumes, and post-earthquake physical damage and restoration patterns, they can be easily evaluated without being involved in analytical complications (Chang and Nojima, 1997).

Most frequently-used measure is "connectivity." Connectivity is defined as reachability of an arbitrary O/D pair via at least one route, which can be easily calculated by performing Boolean operations of the adjacent matrix representing existence or absence of links between all pairs of nodes. However, it is usually optimistic that physical interconnection satisfies a required function of the road network regardless of traffic capacity and trip length.

Definition of maximum flow as a serviceability measure

"Maximum flow," defined as a flow of largest possible value (Dolan and Aldous, 1993), explicitly characterize the serviceability of transportation systems. Suppose that the road network is composed of n links subject to random and independent occurrence of failure. Given a network configuration and a set of flow capacity on each link $\mathbf{C} = \{c_1, c_2, \ldots\ldots c_n\}$, maximum flow can be calculated using Ford-Fulkerson's algorithm based on "the max-flow min-cut theorem." When applied to a specific damage condition of road network, maximum flow is an essential ingredient in determining serviceability of the system (Fenves and Law, 1979). In this study, the system flow capacity, which is defined as the maximum flow between a specific pair of source and terminal nodes, has been adopted as a basic performance measure. Functional degradation can be evaluated on the basis of pre-quake capacity of individual links and post-earthquake structural damage pattern without dealing with confused behavior of emergency traffic.

Binary performance index based on maximum flow

Let binary state variables x_i (i = 1, 2, ... n) denote the state of survival (1) and failure (0) of i-th link, and n-vector $\mathbf{S}$ = {x_1, x_2, ... x_n} denote the overall state of n links. The maximum flow $F_{max}(\mathbf{S})$ of the road network can be calculated for a damage pattern $\mathbf{S}$ using Ford-Fulkerson's algorithm. The maximum flow F_0 in normal situation can be calculated by assigning $\forall x_i = 1$. Next, a binary variable $B(\mathbf{S})$ corresponding to satisfying (1) or unsatisfying (0) a pre-assigned requirement level r $(0 < r \leq 1)$ of normal traffic capacity F_0 is defined to compute the system reliability in the following section.

$$B(\mathbf{S}) = \begin{cases} 0 & (F_{\max}(\mathbf{S}) < rF_0) \\ 1 & (F_{\max}(\mathbf{S}) \geq rF_0) \end{cases} \tag{1}$$

It is obvious that this criterion is equivalent to the physical connectivity for small value of r. Thus, the performance analysis based on the system flow capacity implicitly incorporate the connectivity analysis as a special case.

STATE ENUMERATION AND MONTE CARLO SIMULATION FOR SYSTEM RELIABILITY ANALYSIS

Li's scheme for approximation of system performance measure

Because the actual damage condition $\mathbf{S}$ is unknown before the occurrence of earthquake disaster, probabilistic approach has been employed. Assume that a set of reliability index $\mathbf{p}$ = {p_1, p_2, ... , p_n} = {E[x_i]} (i = 1, 2, ... n) defined as expected values of the binary state valuables x_i is given for each link. Denote the overall state of the system by $\mathbf{S}^k$ (k = 1, 2, 3, ..., 2^n) and the probability of occurrence of each state $\mathbf{S}^k$ by $Q(\mathbf{S}^k)$, and suppose that the performance measure takes on a value $G(\mathbf{S}^k)$ when the network is in state $\mathbf{S}^k$. In this study, $G(\mathbf{S}^k)$ is a general expression representing the maximum flow $F_{max}(\mathbf{S}^k)$ itself or the binary performance measure $B(\mathbf{S}^k)$. The expected value of the performance measure, typically denoted by E[$G(\mathbf{S})$], is derived by averaging $G(\mathbf{S}^k)$ over the possible states with probability $Q(\mathbf{S}^k)$.

$$\mathrm{E}[G(\mathbf{S})] = \sum_{k=1}^{2^n} Q(\mathbf{S}^k)\, G(\mathbf{S}^k) \tag{2}$$

Obviously, in order to obtain the exact value, as many as 2^n terms of possible states must be exhaustively enumerated, which is virtually impossible to perform for large n. To avoid this "state-space explosion," Li et al. (1984, 1986) proposed an efficient algorithm to generate the most probable m ($m < 2^n$) states of the network and evaluate upper and lower bounds of system performance measure, G_U and G_L, respectively. The two bounds are obtained by the equations

$$G_U = \sum_{k=1}^{m} Q(\mathbf{S}^k)\, G(\mathbf{S}^k) + \{1 - \sum_{k=1}^{m} Q(\mathbf{S}^k)\}\, G_b \tag{3}$$

$$G_L = \sum_{k=1}^{m} Q(\mathbf{S}^k) G(\mathbf{S}^k) + \{1 - \sum_{k=1}^{m} Q(\mathbf{S}^k)\} G_w \tag{4}$$

where G_b and G_w represent the performance measure for the best (no components fail) and worst (all components fail) state, respectively. This approximation method, which was originally developed by Li and Silvester (1984) as the algorithm ORDER, and modified by

Lam and Li (1986) as the algorithm ORDER-II, utilizes a binary tree with the priority that the value of each node is no larger than the value of its children nodes. Asakura et al. (1997) applied this scheme to evaluate road network reliability in terms of the shortest route between a specific O/D pair. Generally, this method is advantageous when the number of components n is considerably small and the component reliability p_i take on values close to extremes of 1 or 0, i.e., highly reliable or highly vulnerable. Reliability assessment of communication network in normal situation is one of the most suitable application. On the other hand, the approximation method is hopelessly insufficient for systems composed of large number of components with non-extreme value of reliability, which is common in lifeline network systems under seismic environment; the state space covered by the partial enumeration is so small that the convergence of upper and lower bounds to the exact value is too slow.

Formulation of variance reduction technique in Monte Carlo simulation using Li's bounds

In this study, a hybrid method that combines the Li's partial enumeration technique and the Monte Carlo simulation method is developed for solution of the problem mentioned above. The method proposed herein uses the two bounds as upper and lower limits of probability space for variance reduction in performing Monte Carlo simulation; the sample space is limited to the compliment of the state space enumerated by using Li's algorithm. On the basis of variance reduction technique, the approximate value of the system performance measure is obtained by

$$G(m,N) = \sum_{k=1}^{m} Q(\mathbf{S}^k)\,G(\mathbf{S}^k) + \{1 - \sum_{k=1}^{m} Q(\mathbf{S}^k)\}\sum_{j=1}^{N} G(\mathbf{S}^j)/N \tag{5}$$

where $G(\mathbf{S}^j)$ denotes system performance measure for the state $\mathbf{S}^j$ generated at j-th trial, and N denotes number of trials of simulation. The first term of Eq.(5) is associated with enumeration of the space of the most probable m states, and the second term is random sampling out of the remaining space. Approximate value of the expected performance measure $G(m, N)$ can be obtained with less variance than crude Monte Carlo method. For example, assume that the exact value of system reliability is $P_0 = \mathrm{E}[B(\mathbf{S})]$. Without variance reduction, the variance of the estimator of the system reliability is given by $P_0(1-P_0)/N$. Theoretically, the variance of the estimator with upper and lower bound P_U and P_L is reduced to $(P_U - P_0)(P_0 - P_L)/N$, which realizes precise estimation for narrow boundary space (P_U - P_L) (Henley and Kumamoto, 1981).

This hybrid method is widely applicable without being limited by the size of the network and/or value of reliability. The optimum combination of the number of partial enumeration m and the number of simulation trials N depends on circumstances. Appropriate size of enumeration and simulation must be explored through pilot runs of the proposed scheme.

Birnbaum's probabilistic importance measure

Let $P(\mathbf{p})$ denote the system reliability as a function of the component reliability vector $\mathbf{p}$ = $\{p_1, p_2, ..., p_n\}$. The conditional system reliability given that a specific component i fails or not are written as $P(0_i, \mathbf{p})$ and $P(1_i, \mathbf{p})$, respectively. The difference between these two is referred to as "Birnbaum's probabilistic importance measure," which implies probabilistic contribution of improving component reliability to that of system reliability (Henley and Kumamoto, 1981).

$$I^B{}_i = \frac{\partial P(\mathbf{p})}{\partial p_i} = P(1_i,\mathbf{p}) - P(0_i,\mathbf{p}) \quad (6)$$

The proof of Eq.(6) is very simple. According to the theorem of total probability, the system reliability $P(\mathbf{p})$ can be written as a linear function in terms of p_i.

$$P(\mathbf{p}) = p_i \cdot P(1_i,\mathbf{p}) + (1 - p_i) \cdot P(0_i,\mathbf{p}) \quad (7)$$

Eq.(6) can be obtained by differentiating both sides of Eq.(7) by p_i.

The conditional system reliability $P(0_i, \mathbf{p})$ and $P(1_i, \mathbf{p})$ can be separately computed during the procedure of reliability analysis mentioned above, so the importance measure is easily calculated by use of Eq.(6). Ranking of this measure computed for all the links rationally determines retrofitting prioritization. By upgrading links rated highly important with priority, the system flow capacity can be effectively improved. If cost-effectiveness is appropriately considered, cost allocation for seismic the retrofitting program can be optimized.

NUMERICAL EXAMPLE USING SIMPLE NETWORK MODEL

Network model and assumed conditions

The road network model for a numerical example is shown in Fig.1. The network model is composed of 22 links with three kinds of traffic capacity, 700, 1400 and 2000 (vehicles/hr) and 15 nodes including cities A, B, C and D. Suppose that the cities A, B and C are undamaged and the city D is severely damaged. The demand to the road network is keeping traffic capacity as much as possible to clear the way for emergency operation and logistics supply from A, B and/or C to D. The number attached on each link represents the link ID. The link number 1, 2 and 3 are not shown because they are assigned to artificial links connecting the dummy source node to the actual source nodes A, B and C. Maximum flow in normal condition is 5400 (= 1400 + 2000 + 2000) (vehicles/hr) which is determined by three links (links No.19, 23 and 24) composing a bottleneck around the node D.

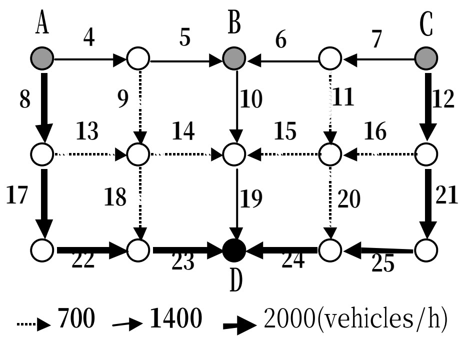

Fig.1 Model of road network.

Many cases have been examined to survey general tendency depending on magnitude of disaster and requirement levels as illustrated in Fig.2. For simplicity, it is assumed that component reliability is uniform over the entire links, and the uniform value is ranging from p = 0.1 to 0.9 in accordance with major to slight disaster. As for requirement level of flow capacity, three different levels are assumed: r = 20, 50, and 80% of the normal capacity. At each level, specifically, the flow capacity 1080, 2700 and 4320 (vehicles/hr) must be satisfied to be considered that the system function is alive. Graphical demonstration is provided below for the case where the number of partial enumeration m = 2000, and the number of simulation trials N = 8000.

Results

Fig.3 shows the ratio of state space covered by 2000 enumeration of most probable states. As noted in the previous section, it is observed that the result is significantly dependent on the component reliability. If p = 0.8 (or equivalently p = 0.2), 83% of the entire probability space is covered by 2000 enumeration, while only 0.05% is covered if p = 0.5. This indicates that in the latter case, Li's method results in only a crude enumeration.

Expected maximum flow F_{max} from nodes A, B and C to D approximated by use of Eq.(5) is shown in Fig.4. As the component reliability p decreases from 1 to 0.5, the value of F_{max} considerably decreases from 5400 (the normal value) to 780, and then gradually approaches to 0 with further decreasing value of p.

The system reliability P computed by use of Eq.(5) is plotted in Fig.5 as a function of component reliability p for various values of requirement level r. Generally, the system reliability is degraded from 1 to 0 as component reliability decreases from 1 to 0. In particular, if the requirement level is r = 80%, i.e., decrease in traffic capacity by more than 20% is unacceptable, the system reliability drops to very low level. Conversely, if the requirement level is r = 20%, i.e., up to 80% decrease of traffic capacity is acceptable, the system reliability remains relatively high. In the latter case, it can be said that small amount of traffic capacity should be reserved by conducting strict control of traffic so as to prioritize emergency activities.

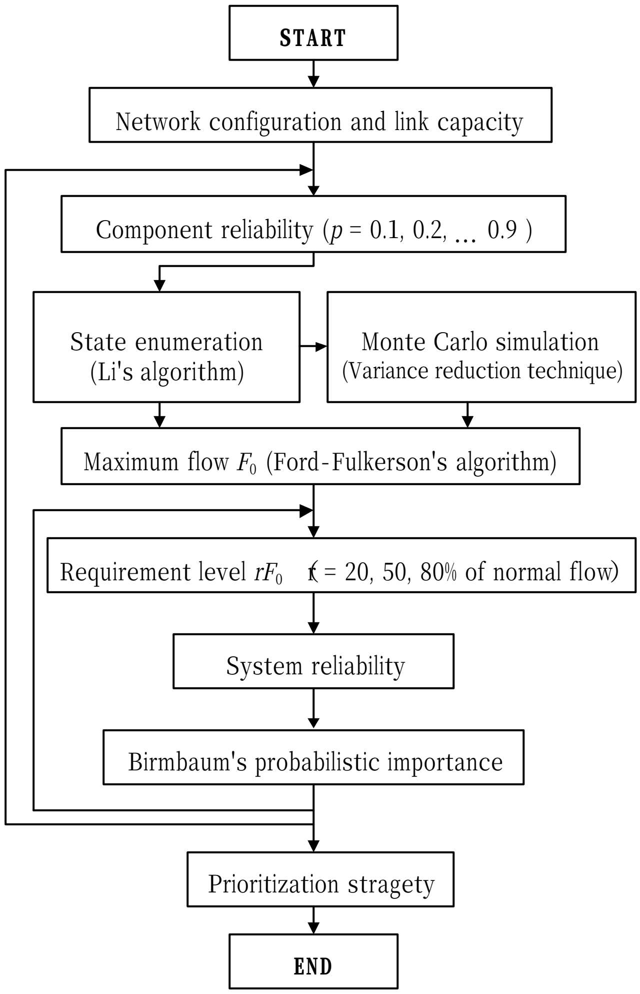

Fig.2 Flow chart of numerical examples.

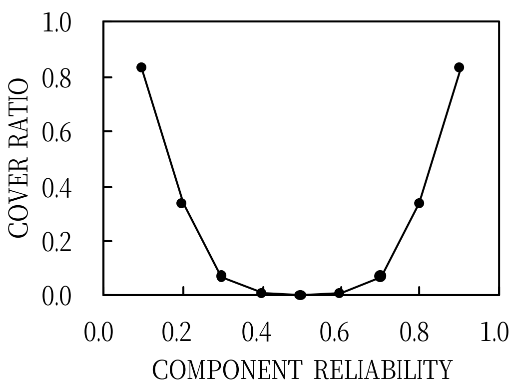

Fig.3 Ratio of state space covered by 2000 enumeration.

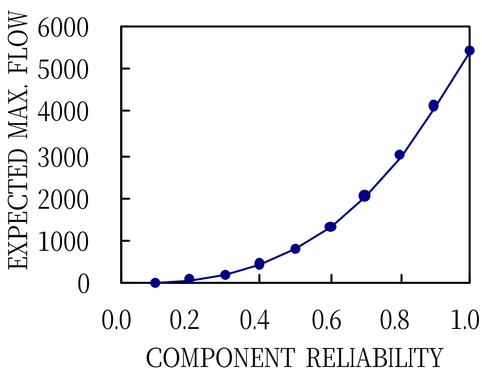

Fig.4 Expected maximum flow as a function of component reliability.

Importance ranking is strongly dependent on the combination of the component reliability p and the requirement level r. Fig.6 shows the conditional system reliability $P(0_i, p)$ and $P(1_i, p)$ when the component reliability is low (p = 0.3) and the requirement level is also low (r = 0.2). The link numbers are ranked according to the probabilistic importance measure I^B_i, i.e., the difference between the solid and open bars for each link. Similarly, the results for high p (= 0.9) and low r (= 0.2) is shown in Fig.7. Fig.8 graphically shows several links of high importance for various combinations of p and r. Major findings derived from Fig.6-8 are listed below.

(1) For low p and low r, as shown in Fig.6, two links (No.10 and 19) that composes the shortest route from the node B to D are rated highly important, because its capacity 1400 (vehicles/hr) satisfies the requirement 1080 (vehicles/hr). This implies the connectivity of road network is of great interest regardless of the flow capacity. Please note that the shortest route theoretically means the most reliable route, when p is uniform over all the links.

(2) For high p and high r, as shown in Fig.7, trunk links with large capacity (links No.23, 25, ..., 19, 12) are ranked high, because any failure of them definitely leads to dissatisfaction of high requirement, in this case, the flow capacity 4320 (vehicles/hr).

(3) For moderate p and moderate r, links of high importance are mixture of the above two extreme cases.

(4) For high p and low r, three links (No.19, 23 and 24) directly connected to node D are found to be of highly important, because these links lack redundancy and potentially composes a bottleneck.

(5) It is obvious that in the case where r is relatively high compared with p, the system reliability is extremely low (P < 0.01). Damage must be accepted in these cases.

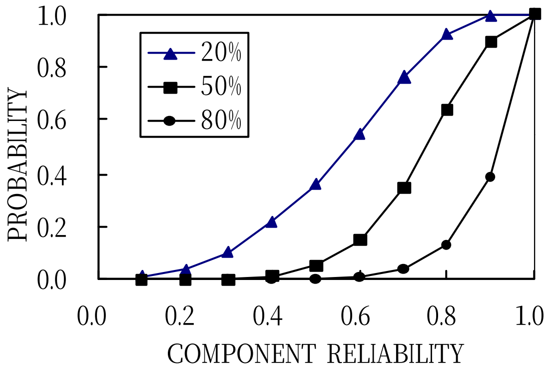

Fig.5 Component reliability and system reliability for various r.

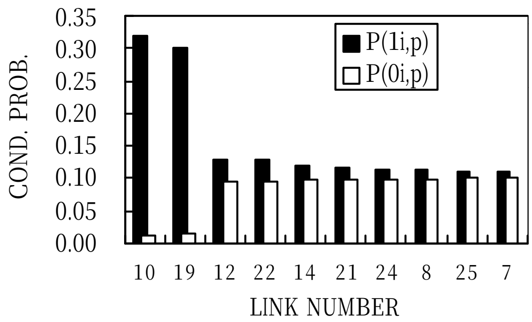

Fig.6 Conditional system reliability given that link i survives or not (p = 0.3, r = 20%).

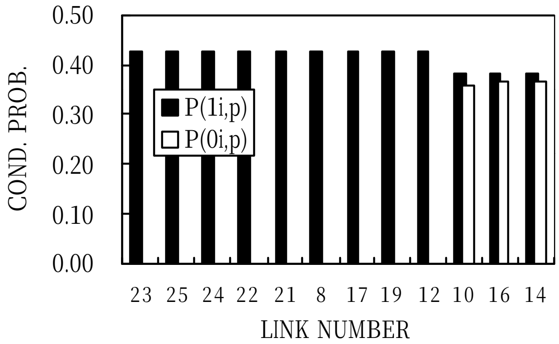

Fig.7 Conditional system reliability given that link i survives or not (p = 0.9, r = 80%).

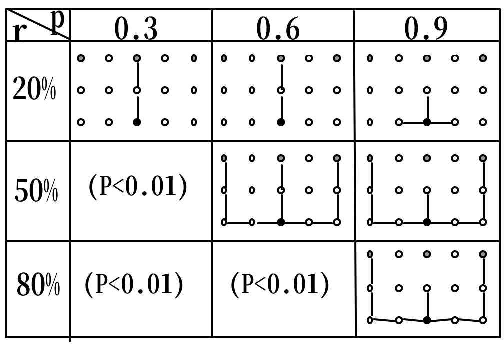

Fig.8 Links of high importance for various p and r.

CONCLUDING REMARKS

A probabilistic method has been proposed to assess the seismic reliability of road networks in terms of maximum flow, and Birnbaum's probabilistic importance measure has been introduced for probabilistic rationale of retrofitting prioritization. The method proposed herein requires only simple information on network properties to determine importance ranking: (1) network configuration of road network systems, (2) a set of pre-earthquake link capacity c_i of all links, (3) a set of component reliability p_i of all links, and (4) post-earthquake requirement level r.

The numerical examples indicate that the importance ranking strongly depends on a combination of anticipated magnitude of disaster and required service level of the system concerned. If enough capacity must be maintained, trunk lines should be prioritized in order to accommodate large traffic flow. On the other hand, the shortest route (or the most reliable route) is of great importance, when the requirement level is appropriately suppressed to preserve emergency traffic, which is possible situation immediately after the earthquake.

Recently, disaster prevention plans involve different sizes and different types of scenario earthquakes in conjunction with regional seismic hazard. Therefore, the concept of “performance-based prioritization” will be useful for strategic implementation of structural retrofitting program.

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