Partial factors calibration based on reliability analyses for square footings on granular soils

Y. HONJO* and S. AMATYA*

Partial safety factors for square footings for highway bridges resting on granular soils have been determined based on reliability analyses. Some example cases are chosen based on a database which includes detailed information of 1869 actually constructed highway bridge pier shallow foundations in one fiscal year in Japan. The designs obtained, using calculations based on the bearing capacity equations by Meyerhof and by Brinch Hansen as modified by Vesic, are compared. The uncertainties involved in the bearing capacity equations are investigated through a comprehensive literature review. The seismic forces determined from the peaks over threshold analysis and fitted to a general Pareto distribution have been considered. The first-order reliability method (FORM) and Monte Carlo simulations (MCS) are employed to determine and compare the 100-year failure probabilities of the shallow foundations designed. It is found that FORM gives a considerably lower failure probability than MCS. Finally, partial factors obtained using calculations based on the two bearing capacity equations are carefully reviewed. It is found that the design value method used to determine partial factors by FORM does not appropriately give either the partial factors or the failure probability for the case of shallow foundation design where the performance function is highly non-linear and some of the basic variables follow distributions that are far from the normal distribution.

KEYWORDS: design; footings/foundations; statistical analysis

INTRODUCTION

LSD and LRFD

The limit state design (LSD) method, or equivalently the load and resistance factor design (LRFD) method, has been accepted as the standard basis on which geotechnical design codes are to be developed today. In Europe (Eurocode 7, 1994), Canada (Becker, 2003), China (Zhang, 2003), Japan (Honjo et al., 2000; Okahara et al., 2003), the USA (Kulhawy & Phoon, 2002; Withiam, 2003) and elsewhere, major geotechnical design codes are switching from allowable stress design (ASD), or equivalently working stress design (WSD), to LSD and LRFD. The development and implementation of LSD and LRFD have been driven primarily by the objectives of achieving a consistent design philosophy so as to bridge structural and geotechnical engineering, and obtaining a more consistent and rational framework of risk management in geotechnical engineering. Les facteurs partiels de sécurité pour les assises carrées de ponts d’autoroutes reposant sur des sols granulaires ont été déterminés sur la base d’analyses de fiabilité. Certains cas cités en exemples sont choisis dans la base de données contient une information détaillée sur 1869 fondations peu profondes de piles de pont routier construites en une année fiscale au Japon. Nous comparons les plans obtenus, utilisant des calculs basés sur les équations de capacité porteuse de Meyerhof et de Brinch Hansen modifiées par Vesic. Nous étudions les incertitudes qui apparaissent dans les équations de capacité porteuse en faisant une révision poussée de la documentation. Nous prenons en compte les forces sismiques déterminées à partir des analyses crêtes sur seuil et correspondant à une distribution General Pareto. Nous employons la méthode de fiabilité de premier ordre (FORM) et les simulations de Monte Carlo (MCS) pour déterminer et comparer les probabilités de rupture sur un cent ans des fondations peu profondes ainsi conçues. Nous trouvons que FORM donne une probabilité de rupture considérablement plus basse que MCS. Enfin, nous révisons les facteurs partiels obtenus en utilisant les calculs basés sur les deux équations de capacité porteuse. Nous trouvons que la méthode de valeur nominale utilisée pour déterminer les facteurs partiels au moyen de FORM ne donne de manière appropriée ni les facteurs partiels, ni la probabilité de rupture pour le cas de fondations peu profondes lorsque la fonction performance est extrêmement non linéaire et certaines des variantes de base suivent des distributions qui sont éloignées de la distribution normale.

Design verification format

It is identified by many that key issues in developing sound geotechnical design codes based on LSD and LRFD are the definition of characteristic values and the determination of partial factors together with the formats of design verification (e.g. Simpson & Driscoll, 1998; Honjo & Kusakabe, 2002; Kulhawy & Phoon, 2002; Orr, 2002). The definition of characteristic values of geotechnical parameters is out of the scope of this paper and thus is excluded from the discussion here. The characteristic values of all the basic variables, except the seismic load, are conveniently defined as their mean values in this study. The characteristic value of the seismic load is assumed to be that with a 100-year return period.

The main argument concerning design verification formats can be summarised as whether one should take a material factor approach (MFA) or a resistance factor approach (RFA). In MFA, partial factors are applied directly to characteristic values of materials in design calculations, whereas in RFA a resistance factor is applied to the resulting resistance calculated using the characteristic values of materials. One of the modifications of RFA is a multiple resistance factor approach (MRFA), in which several resistance factors are employed to be applied to relatively large masses of calculated resistances. The advantage of MRFA is
that it is claimed to ensure a more consistent safety level for a variety of design conditions compared with RFA (Phoon et al., 1995, 2000; Kulhawy & Phoon, 2002). It is a well-known fact that MFA originated from Europe, whereas RFA came from North America. However, they are now used interchangeably worldwide.

There is no one agreed rule or tradition regarding whether MFA or RFA should be employed in design verification formats (Simpson, 2000). The advantages of RFA and MRFA over MFA may be listed as follows.

(a) RFA and MRFA allow designers to get a ‘feel’ of the actual behaviour of their design up to the last stage of their design work. This aspect is more important in geotechnical design, where the interactions of a structure and the ground are very complex, and a reduction (or increase) in the soil parameter values may not always introduce more safety to the design. For example, in the design of a laterally loaded pile, a reduction of the horizontal subgrade reaction coefficient may lead to an increase in deformation, whereas a larger value may result in an increase in the stress in the pile. Apparently, this aspect would be more significant when more sophisticated design calculation methods, such as finite element methods, are introduced.

(b) When one investigates the uncertainties in a design calculation, it often happens that only total results are comparable between the calculated values and the true values, for example the calculated total pile capacity and the results of pile loading tests. In such cases, only overall uncertainty can be quantified for use in a reliability analysis. This fact implies that it is more reasonable to carry out calibration with RFA than with MFA.

(c) RFA and MRFA are closer to the traditional geotechnical design format, and thus are considered to cause less confusion among practising geotechnical engineers when implemented.

On the other hand, MFA is considered to be better in the following aspects compared with RFA.

(a) It is natural, at least intuitively, to take care of uncertainties at their sources.

(b) It gives more flexibility to the magnitude of the safety level introduced at each source of uncertainty. For example, it is easier for code writers to change partial factors when a new construction method is developed: one can change the partial factors corresponding to the position where the improvements have taken place.

**Code calibration**

A procedure to rationally determine partial factors in the design verification equations based on reliability analysis is termed code calibration. One of the best-known works in this area is by Ellingwood et al. (1982), wherein load and resistance factors are determined based on a reliability analysis using FORM. Since then, a reasonable number of code calibration works have been carried out in structural engineering. However, rational code calibration works in geotechnical engineering codes started only in the past decade or so (e.g. Barker et al., 1991; Phoon et al., 1995; Honjo et al., 2002; Paikowsky et al., 2004).

Barker et al. (1991) proposed resistance factors for the AASHTO bridge foundation code published in 1994 (AASHTO, 1994). The calibration was based on first order second moment (FOSM) but introduced a significant amount of engineering judgement in determining the factors, and the process was not very clearly described. Based on the reflections on their work, the partial factors on the deep foundations in the AASHTO specification were revised by Paikowsky et al. (2004), in which a large database was developed and used together with reliability analysis by FORM to determine the factors. Phoon et al. (1995, 2000) calibrated the factors for transmission line structure foundations based on MRFA by reliability analysis. Some simplified design formats were employed, and factors were adjusted until the target reliability index was reached. Kobayashi et al. (2003) have calibrated resistance factors for building foundations for the AIJ limit state design building code (AIJ, 2002). This code provides a set of load and resistance factors for all aspects of building design in a unified format. FORM was used for the reliability analysis, and MRFA was the adopted format for design verification as far as the foundation design was concerned.

**Scope of this study**

An attempt is made in this paper to determine the partial safety factors for shallow foundation designs for highway bridges. Square footings resting on the surface of cohesionless soils are considered. The reason why only square footings are considered is that most of the shallow foundations used in highway bridges are close to square footings.

The uncertainties involved in the bearing capacity equations have been investigated through a comprehensive literature review. Proposals made by Meyerhof (1963) and Brinch Hansen (1970) as modified by Vesic (1975) (hereafter referred to as Vesic, 1975) are adopted for computing the bearing capacities of shallow foundations. In actual practice in Japanese bridge foundation design, the Specifications for Highway Bridges (JRA, 1996) of the Japan Road Association use a modified version of Meyerhof’s equations.

The parameters considered for partial factor application in the bearing capacity estimation are the bearing capacity factor $N_p$, the modification factor for the inclination of loading, $i_p$, and Meyerhof’s effective width, $B_e$. Ingra & Baecher’s (1983) results have been used mainly to quantify the uncertainties in the determination of these factors. The partial factors for dead (i.e. permanent) and seismic loads are also determined in the code calibration. The uncertainty of the seismic forces, estimated by POT (peaks over threshold) analysis and fitted to a General Pareto distribution, has been considered.

The probability of failures and the reliability indexes for some example cases are obtained from FORM analyses as well as from Monte Carlo simulations.

**EVALUATION OF UNCERTAINTIES**

**Uncertainties in loads**

Annual maximum ground surface acceleration. Seismic load is the dominant source of uncertainty in Japan, and the distribution parameters for it have been determined using POT (peaks over threshold) analysis. Seismic accelerations for Tokyo for the years 1600–1995 have been extracted from the Usami catalogue (Usami, 1997). Then the maximum ground surface acceleration for each earthquake at a concerned point has been estimated using the Fukushima–Tanaka (1991) attenuation model. Fig. 1 shows the estimated yearly maximum accelerations obtained for a point in Tokyo during the period from 1600 to 1995 (Honjo & Amatya, 2001).

The data presented in Fig. 1 were analysed by POT analysis. POT analysis is a relatively new method in extreme statistics to fit data to extreme distributions. It is based on the finding that a conditional distribution given a threshold value $u$, that is $F(x|u) = [F(x) - F(u)]/[1 - F(u)]$, of data
following the extreme distributions, \( F(x) \), follows the Generalized Pareto (GP) distribution (Balkema & de Haan, 1974; Pickands, 1975). Based on this finding, it becomes possible to fit only the tail part of the given data to the GP distribution to estimate the extreme events. A program has already been developed (Reiss & Thomas, 1997), which was used in this study.

The annual distribution function for the maximum ground acceleration, \( S_e \), can be arrived at by the following equation from the POT analysis:

\[
F(S_e) = \frac{n + 1 - k}{n + 1} - \frac{k}{n + 1} \left[ 1 + \left( 1 + \frac{S_e - \mu_S}{\sigma_S} \right)^{-1/\gamma_S} \right]^{1/(n + 1)}
\]  

(1)

This is valid for \( S_e \geq \mu_S \). Here, \( n \) is the number of data, \( k \) is the number of data exceeding a threshold value (\( \mu_S \), cm/s\(^2\)), and \( \gamma_S \), \( \mu_S \), and \( \sigma_S \) are respectively the shape, location and scale parameters of the GP distribution.

For the POT analysis, the threshold seismic acceleration values are fixed by using exploratory data analysis tools such as sample mean excess function (mef) plots, \( \gamma_S \)-estimate plots (here \( \gamma_S \) is the shape parameter of the GP distribution; not to be confused with the partial factor for \( S_e \)) using different estimating methods—maximum likelihood estimate (MLE), moment estimate and Drees–Pickands estimate—and qq-plots (Honjo & Amatya, 2001). Fig. 2 shows the estimates for \( \gamma_S \) against \( k \), and Fig. 3 is the mean excess plot. The threshold values are so chosen that the \( \gamma_S \) values lie in the portions where the sample mef ‘looks linear’ (Bassi et al., 1998) and the number of exceedances, \( k \), lies in the ‘plateau’ of the \( \gamma_S \)-estimate plot.

The distribution parameters thus estimated have been found to be satisfactory and stable for the calculations of maximum accelerations of up to a 500-year return period.

The 100-year maximum seismic force distribution function. A 100-year reference period for a structure is chosen for the calculation of seismic load. For the conversion of seismic acceleration to seismic force, the relationship of proportions given in the following equation has been used:

\[
G_{100}(V_e) = G_{100} \left( \frac{S_e}{S_{e100}} \right) = F_{100} \left( \frac{V_e}{V_{e100}} - S_{e100} \right) = F_1 \left( \frac{V_e}{V_{e100}} - S_{e100} \right)^{100}
\]

(2)

where \( G_{100}(V_e) \) is the 100-year maximum seismic force distribution function for seismic force \( V_e \), \( S_e \) is the maximum ground acceleration for which \( V_e \) is to be calculated, \( V_{e100} \) is the seismic force employed in the current design specification (a characteristic value of \( V_e \)) that is assumed to be generated by the 100-year return period acceleration \( S_{e100} \). \( F_{100} \) is the 100-year maximum ground acceleration distribution function, and \( F_1 \) is the annual maximum ground acceleration distribution function, which is given by equation (1).
The determination of the seismic force $V_{e100}$ is dealt with in detail below in the section ‘Design cases’.

Uncertainties in bearing capacity

**Theoretical solutions.** As proposed by Terzaghi (1943), for a case of central vertical loading on a rigid strip footing on a horizontal soil surface with a uniform surcharge of $q$, the ultimate bearing capacity is given by

$$q_u = cN_c + qN_q + \frac{1}{2} \gamma BN_f$$  \hfill (3)

where $c$ is the cohesion of the soil, $\gamma$ is the unit weight of the soil, $B$ is the breadth of the footing, and $N_c$, $N_q$ and $N_f$ are bearing capacity factors for cohesion, surcharge and self-weight of soil respectively.

Exact closed-form solutions for $N_c$ and $N_q$ are obtainable using the limit analyses:

$$N_c = (N_q - 1) \cot \phi$$

$$N_q = \exp(\pi \tan \phi) \cdot \tan^2 \left( \frac{\pi}{4} + \frac{\phi}{2} \right)$$  \hfill (4)

But no closed-form solution is available for $N_f$, and there are a number of proposals for its calculation:

Vesic (1973):

$$N_f = 2(N_q + 1) \tan \phi$$  \hfill (5)

Meyerhof (1963):

$$N_f = (N_q - 1) \tan(1 - 4\phi)$$  \hfill (6a)

Brinch Hansen (1970):

$$N_f = 1.5(N_q - 1) \tan \phi$$  \hfill (6b)

Chen (1975):

$$N_f = 2.0(N_q + 1) \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right)$$  \hfill (6c)

Ingra & Baecher (1983):

$$N_f = \exp(-2.046 + 0.173\phi) \text{ for } L/B = 1.0$$  \hfill (6d)

Eurocode 7 (1994):

$$N_f = 2.0(N_q - 1) \tan \phi$$  \hfill (6e)

Michalowski (1997):

$$N_f = \exp(0.066 + 5.11 \tan \phi) \tan \phi$$  \hfill (6f)

The plots for these equations are presented in Fig. 5 for a quick comparison.

To deal with a greater variety of loading cases, the following modification was proposed by Brinch Hansen (1961) to include the effects of the shape of footing, inclination and eccentricity of load, and depth of footing on bearing capacity:

$$q_u = s_i c_i d_c c N_c + s_i l_i d_q q N_q + \frac{1}{2} s_i l_i f_l \gamma B' N_f$$  \hfill (7)

where the factors $s$ are shape factors, $i$ are load inclination factors, and $d$ are footing depth factors. $B'$ is the effective width of the footing calculated for the consideration of load eccentricity. There are a number of proposals for the calculation of these modification factors, for example Meyerhof (1963), Brinch Hansen (1970) and others as listed in Zadroga (1994).

As has been summarised by Siddiquee et al. (2001), $i_f$ for a load inclined at $\alpha$ to the vertical given by Meyerhof (1963) as modified by Vesic (1975), Meyerhof & Koumoto (1987) is

$$i_f = \cos \alpha \left( 1 - \frac{\sin \alpha}{\sin \phi} \right)$$  \hfill (8)

It has been found by the same authors that the proposed equations given by Brinch Hansen (1970) and Gotti & Butterfield (1993) are not consistent with that of Meyerhof. As compared with these two proposed equations, equation (8) underestimates the negative effects of load inclination, a trend that increases with increase in the load inclination. Hence the bearing capacities obtained in this study are overestimated ones.

For determining $B'$, Meyerhof’s rule of effective width for a footing load with an eccentricity of $\epsilon$ will be put into use. According to this rule, the bearing capacity will be calculated for an effective width given by

$$B' = B - 2e = B - 2(M/V)$$  \hfill (9)

where $M$ is the moment loading generated by the seismic acceleration $S_m$, and $V$ is the total vertical load equal to the sum of $V_e$, the dead (permanent) load, and $V_c$, the vertical component of the seismic load. These are discussed further in the section ‘Design cases’.

This hypothesis has been sometimes criticised as being over-conservative. Michalowski & You (1998) have found that it yields a bearing capacity equivalent to that calculated based on the assumption that the footing is smooth. It gives reasonable results for any type of soil–footing interfaces for small eccentricities ($\epsilon/B < 0.1$) and for cohesive or cohesive-frictional soils when the soil–footing interface is not bonded. It underestimates the bearing capacity of a footing on cohesive soils (with a margin of about 8%) with frictional or adhesive soil–footing interfaces, whereas it overestimates for purely frictional soils when the surcharge load is relatively small.

Further sources of uncertainty are added by the influences of the non-associativity of plastic soil deformation, dilatancy and mobilised friction angle; progressive failure phenomenon; scale effect; footing base roughness; initial void ratio; crushing of soil particles; and anisotropy of soil.

Uncertainties quantification. Because of all the complicated factors and interactive effects that influence the resulting bias and uncertainties on bearing capacity prediction, the work by Ingra & Baecher (1983) is mainly relied on to evaluate the uncertainties. They have evaluated the uncertainties in the bearing capacity predictions inferred through statistical analyses of experimental results of prototype footings and model tests compiled from different authors. The ratios of the bearing capacities predicted by the
proposed equations given by Ingra & Baecher to those predicted by the equations given by Meyerhof (1963) and Brinch Hansen as modified by Vesic (1975) (tabulated in Table 1) have been used to incorporate uncertainties in the performance function.

(a) Uncertainty in $N_r$. The $N_r$ for square footings proposed by Ingra & Baecher (1983) from the regression analysis of experimental data, $E[N_r]$, has been considered as the reference value. The ratio $\delta_{N_r}$, defined as the ratio between $E[N_r]$ and $N_r$ in Table 1, has been used to incorporate the uncertainty in $N_r$, which is given as

$$E[N_r] = \exp(-2.064 + 0.173\phi)$$

$$V[N_r] = 0.0902 \exp(-4.128 + 0.346\phi)$$  \hspace{1cm}(10)

(b) Uncertainty in $i_r$. The ratio $\delta_{i_r}$, defined as the ratio between $E[i_r]$ proposed by Ingra & Baecher (1983) and $i_r$ in Table 1, is taken to incorporate uncertainties in the calculation of the effect of inclined loading, where

$$E[i_r] = 1 - 0.41(H/V) + 1.36(H/V)^2$$

$$V[i_r] = 0.0089$$  \hspace{1cm}(11)

(c) Uncertainty due to load eccentricity. Michalowski & You (1998) found that Meyerhof’s effective width $B' = B - 2e$ overestimates the bearing capacity for purely frictional soils when the surcharge load is relatively small. The ratio $\delta_i$ defined below is taken to incorporate the uncertainties in Meyerhof’s effective width:

$$B' = B - 2e$$

$H = \text{horizontal component of seismic load; } V = V_d + V_e = \text{vertical component of load (} V_d = \text{dead load, } V_e = \text{vertical component of seismic load); } \alpha = \text{inclination of load to the vertical } = \tan^{-1}(H/V); A = \text{effective contact area } = B' \times L; \ L = \text{effective width of footing, L = length of footing}; \ e = \text{unit adhesion on footing base (taken as 1)}; m = (2 + B/L)/(1 + B/L); N_e = \exp(\pi\tan(\phi)N_q); N' = \tan^2(45 + \phi/2). \ All \ angles \ are \ in \ degrees.$


<table>
<thead>
<tr>
<th>Design variable considered for reliability analysis and uncertainties in the design parameters</th>
<th>Distribution followed</th>
<th>Distribution parameters</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy of bearing capacity mod. factor $i_r$, $\delta_{i_r}$</td>
<td>Normal</td>
<td>$E[i_r]_{\text{Ingra}}$</td>
<td>Ingra &amp; Baecher (1983)</td>
</tr>
<tr>
<td>Accuracy of bearing capacity factor $N_r$, $\delta_{N_r}$</td>
<td>Normal</td>
<td>$E[N_r]_{\text{Ingra}}$</td>
<td>Ingra &amp; Baecher (1983)</td>
</tr>
<tr>
<td>Accuracy of Meyerhof’s effective width $B'$, $\delta_i$</td>
<td>Normal</td>
<td>$B - E[i_r]_{\text{Ingra}}$</td>
<td>JRA (1989)</td>
</tr>
<tr>
<td>Vertical load, $V_d$</td>
<td>Normal</td>
<td>$1.0076$</td>
<td>Honjo &amp; Amatya (2001)</td>
</tr>
<tr>
<td>Seismic acceleration, $S_e$</td>
<td>General Pareto</td>
<td>$0.067$</td>
<td></td>
</tr>
<tr>
<td>COV for $V_d$</td>
<td></td>
<td>$0.015$</td>
<td></td>
</tr>
</tbody>
</table>

Cases of angle of friction: $32^\circ, 35^\circ, 39^\circ$ and $43^\circ$.
Cases of dead loads: $10,000 \text{ kN}, 15,000 \text{ kN}, 20,000 \text{ kN}, 25,000 \text{ kN}, 30,000 \text{ kN}$ and $35,000 \text{ kN}$.
Unit weight of soil: $17.7 \text{ kN/m}^3$.
For seismic acceleration: no. of data = 396; no. of exceedances = 95; $S_{100} = 169 \text{ cm/s}^2$.
Distribution parameters for different distributions are: General Pareto distribution, U1 = shape parameter, U2 = location parameter, U3 = scale parameter; Normal distributions, U1 = mean and U2 = standard deviation for $\delta_{i_r}$, $\delta_i$ and $\delta_{N_r}$ and COV for $V_d$. $\delta_{N_r}$:

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Meyerhof</th>
<th>Vesci</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Std dev</td>
<td>Mean</td>
</tr>
<tr>
<td>$32^\circ$</td>
<td>1.426</td>
<td>0.439</td>
</tr>
<tr>
<td>$35^\circ$</td>
<td>1.456</td>
<td>0.437</td>
</tr>
<tr>
<td>$39^\circ$</td>
<td>1.398</td>
<td>0.420</td>
</tr>
<tr>
<td>$43^\circ$</td>
<td>1.262</td>
<td>0.379</td>
</tr>
</tbody>
</table>
\[ \delta_e = \frac{B[E(e)]}{B - 2e} \]  
where \( E(e) \) has been proposed by Ingra & Baecher (1983) as

\[ E(e) = 1.0 - 3.5(e/B)^2 + 3.03(e/B)^2 \]

\[ V[\phi] = 0.0058 \]  

(d) Uncertainty in conversion of the standard penetration test for the N value (SPT-N) to friction angle. The Specifications for Highway Bridges (JRA, 1996) give the following relation:

\[ \phi = 15 + \sqrt{15N} \text{ for } \phi \leq 45^\circ \text{ and } N > 5 \]

Based on statistical analysis of experimental results from different authors, Fukui et al. (2002) found that the coefficient of variation (COV) of the \( \phi \) values thus obtained was about 8%.

Unfortunately, it was found that not enough research has been carried out concerning the uncertainty in the determination of the modification factor for the footing shapes. Thus this aspect was discarded from the present study, and only square footings are considered.

**Performance function**

After taking the uncertainties into account, the performance function to be evaluated by the reliability analyses is given as follows:

\[ g(\cdot) = \frac{1}{\pi} \delta_s \delta_p \delta_r \delta_N \delta_p \delta_e (B - 2e) - \frac{V_d + V_e}{B(B - 2e)} \]

where \( \delta_s = 1 \) for footings resting on the ground surface, \( V_d \) is the dead (permanent) load, and \( V_e \) is the vertical component of the seismic load, which follows the distribution obtained in equation (2).

The uncertainties quantified and considered in the reliability analyses are summarised in Table 2 for quick reference.

**RELIABILITY ANALYSIS**

**Design cases**

The chosen load and soil parameter design cases have been based on a database by Fukui et al. (1997) on 1869 highway bridge pier shallow foundations that were actually built in a financial year in Japan. Out of these 1869 foundations, there are 331 shallow foundation cases. The soil properties and loading conditions used for designs for both normal and seismic conditions have been listed. The seismic
conditions are assumed to be generated by the 100-year return period acceleration $S_{e100}$ obtained from the POT analysis above. Based on this database, the frequency of highway bridge shallow foundations with particular soil conditions is investigated, and some typical example design cases have been chosen for code calibration: that is, partial factor determination.

In Fig. 6, frequencies of the different friction angles $\phi$ based on the database are plotted, and four cases of $\phi$, 32°, 35°, 39° and 43°, which are the most frequent (i.e. commonly encountered) friction angles in this database, have been chosen for analysis.

Most of the shallow foundations are rectangular footings, for which the length-to-breadth ratios are nearly equal to 1:1. The vertical dead (permanent) loads $V_d$ (kN/m) and the seismic loads $V_e$ (kN/m) are plotted in Fig. 7. The loads are per unit length of the shallow foundations, and clearly show the relationship $V_{e100} = V_d/1.1$. Based on these results, six combinations of $(V_d, V_{e100})$ (kN), namely (10 000, 9091), (15 000, 13 636), (20 000, 18 182), (25 000, 22 727), (30 000, 27 273) and (35 000, 31 818), are chosen, as these loadings cover most of the example data in Fukui et al.’s database.

The loading causing inclination, that is horizontal loads $H$, and load eccentricities or moment loads $M$ are assumed here based on their relationship with the vertical seismic loads, $V_e$, which are presented in Figs 8(a) and 8(b) respectively. The regression lines $H = (0.25 \pm 0.1)V_e$ and $M = (2.2 \pm 0.5)V_e$ are obtained from the data. Here $H$ (kN/m), $V_e$ (kN/m) and $M$ (kN m/m) are respectively the horizontal

| Table 3. Sensitivity factors obtained through FORM analysis |
|-----------------|-----------------|-----------------|
|                | Meyerhof (1963) | Vesc (1975)     |
| $\delta_{K_d}$ | 0.260           | 0.268           |
| $\delta_{K_e}$ | 0.947           | 0.935           |
| $\delta_{\gamma}$ | 0.052         | 0.036           |
| $\delta_{\sigma}$ | 0.002         | 0.002           |
| $\gamma$ | 0.0752           | 0.0752           |
| $\sigma$ | -0.151           | -0.151           |
Table 4. Partial factors obtained based on FORM analysis and probabilities of failure obtained through Monte Carlo simulations (MCS; 1 million simulations for each design case) and FORM

(a) Meyerhof (1963) equation

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
<th>$\gamma_y$</th>
<th>$\gamma_{y_e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of all cases</td>
<td>Mean 0.96 0.63 1.00 1.00 1.21 0.227 0.109 0.337 0.254</td>
<td>Std dev 0.0108 0.0961 0.0012 $7.6 \times 10^{-5}$ 0.0504 0.1065 0.0667 0.1426 0.2183</td>
<td>COV 0.0113 0.1529 0.0012 $7.6 \times 10^{-5}$ 0.0416 0.469 0.614 0.423 0.858</td>
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<tr>
<td>Mean of 32°</td>
<td>Mean 0.95 0.54 1.00 1.00 1.26 0.129 0.0531 0.208 0.103</td>
<td>Std dev 0.0105 0.0272 0.0002 $4.9 \times 10^{-5}$ 0.0247 0.0342 0.0108 0.0569 0.0328</td>
<td>COV 0.0111 0.0508 0.0002 $4.9 \times 10^{-5}$ 0.0197 0.264 0.204 0.273 0.318</td>
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<tr>
<td>Mean of 35°</td>
<td>Mean 0.95 0.57 1.00 1.00 1.24 0.166 0.0683 0.260 0.140</td>
<td>Std dev 0.0102 0.0309 0.0004 $5.4 \times 10^{-5}$ 0.0289 0.0435 0.0145 0.0721 0.0472</td>
<td>COV 0.0107 0.0540 0.0004 $5.4 \times 10^{-5}$ 0.0234 0.263 0.212 0.278 0.336</td>
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<tr>
<td>Mean of 39°</td>
<td>Mean 0.96 0.64 1.00 1.00 1.20 0.242 0.106 0.365 0.239</td>
<td>Std dev 0.0091 0.0383 0.0008 $6.0 \times 10^{-5}$ 0.0336 0.0579 0.0241 0.0883 0.0855</td>
<td>COV 0.0095 0.0597 0.0008 $6.0 \times 10^{-5}$ 0.0280 0.240 0.227 0.242 0.358</td>
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<tr>
<td>Mean of 43°</td>
<td>Mean 0.97 0.77 1.00 1.00 1.15 0.372 0.207 0.516 0.535</td>
<td>Std dev 0.0071 0.0513 0.0012 $4.9 \times 10^{-5}$ 0.0316 0.0675 0.0494 0.0980 0.2564</td>
<td>COV 0.0073 0.0669 0.0012 $5.0 \times 10^{-5}$ 0.0275 0.182 0.238 0.190 0.479</td>
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(b) Vesic (1975) equation

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<td>Mean 0.96 0.66 1.00 1.00 1.23 0.229 0.116 0.378 0.273</td>
<td>Std dev 0.0098 0.0367 0.0006 $5.9 \times 10^{-5}$ 0.0360 0.0536 0.0277 0.0824 0.0974</td>
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<tr>
<td>Mean of 32°</td>
<td>Mean 0.96 0.66 1.00 1.00 1.26 0.210 0.114 0.347 0.242</td>
<td>Std dev 0.0097 0.0269 0.0001 $4.1 \times 10^{-5}$ 0.0229 0.0357 0.0198 0.0609 0.0561</td>
<td>COV 0.0101 0.0406 0.0001 $4.1 \times 10^{-5}$ 0.0182 0.170 0.176 0.176 0.232</td>
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<tr>
<td>Mean of 35°</td>
<td>Mean 0.96 0.65 1.00 1.00 1.25 0.206 0.105 0.346 0.235</td>
<td>Std dev 0.0099 0.0289 0.0002 $4.4 \times 10^{-5}$ 0.0254 0.0386 0.0203 0.0630 0.0578</td>
<td>COV 0.0104 0.0447 0.0002 $4.4 \times 10^{-5}$ 0.0203 0.188 0.193 0.182 0.246</td>
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<tr>
<td>Mean of 39°</td>
<td>Mean 0.96 0.65 1.00 1.00 1.23 0.223 0.109 0.373 0.260</td>
<td>Std dev 0.0101 0.0352 0.0003 $4.9 \times 10^{-5}$ 0.0296 0.0479 0.0251 0.0753 0.0793</td>
<td>COV 0.0106 0.0543 0.0003 $4.9 \times 10^{-5}$ 0.0241 0.215 0.232 0.202 0.305</td>
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</tr>
<tr>
<td>Mean of 43°</td>
<td>Mean 0.96 0.69 1.00 1.00 1.20 0.277 0.137 0.446 0.355</td>
<td>Std dev 0.0097 0.0394 0.0006 $5.4 \times 10^{-5}$ 0.0325 0.0574 0.0320 0.0863 0.1276</td>
<td>COV 0.0101 0.0573 0.0006 $5.4 \times 10^{-5}$ 0.0272 0.207 0.233 0.194 0.359</td>
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</tbody>
</table>

Numbers in parentheses are reliability indexes, $\beta$.

Initial $P_t$ = probability of failure calculated for designs using global factor of safety.

Final $P_t$ = probability of failure calculated for designs using partial factor of safety obtained as the mean of all cases.

load, vertical seismic load and moment per unit length of the foundation. Three cases of $M$, namely $0.35V_e$, $0.25V_e$ and $0.15V_e$, and three cases of $M$, namely $2.7V_e$, $2.2V_e$ and $1.7V_e$, are assumed. The inclinations and eccentricities are calculated based on the equations presented in Table 1. The footings are thus designed for the eccentricities of 0.81m, 1.05m and 1.29m. The inclination angles obtained are arc tan of 0.0714, 0.1190 and 0.1667.

In summary, 216 (= 4 × 6 × 3 × 3) cases have been chosen, which consist of the following:

(a) From the data for the bridge foundations, it can be inferred that the four most commonly encountered angles of internal friction were 32°, 35°, 39° and 43° (Fig. 6).

(b) In accordance with the load intensity range for the majority of the data in Fig. 7, vertical dead (permanent) loads, $V_d$, of 10 000, 15 000, 20 000, 25 000, 30 000 and 35 000 kN have been chosen.

(c) For each $V_d$, $V_{c100}$ is estimated based on the relationship $V_{c100} = V_d/1.1$ (Fig. 7). This only represents the relationship between the characteristic values of the random variables $V_d$ and $V_c$. The variables $V_d$ and $V_c$ are independent of each other.

(d) Horizontal loads and moments are chosen for each $V_c$ according to Fig. 8.

Reliability analyses and partial factors

After the uncertainties and design cases are chosen, the target reliability index needs to be fixed based on the current safety level. The reliability analyses by FORM and MCS are carried out for the 216 cases listed in the previous section designed in the conventional way using a global safety factor.
approximation at the design point causes some serious errors in the estimation of \( \beta \)'s in such cases (e.g. Ditlevsen & Madsen, 1996, pp. 87–94).

**Partial factors and consistent reliability index**

For partial factors \( \gamma \) with appropriate subscripts, applied to the design variables \( i, N, e, B_e \) and \( V_e \), the performance function given in equation (14) can be rewritten as a design verification equation as

\[
g(S, \gamma_i, i, \gamma_N, N, \gamma_eB_e, \gamma_eV_e) = \frac{1}{2} \sigma_S \cdot d_S \cdot \gamma_i \cdot i \cdot \gamma_N \cdot N \cdot \gamma_e \cdot (B - 2e) - \gamma_eV_e + \gamma_eV_e \geq 0
\]

(15)

As summarised in Table 3, the performance function has the highest sensitivity factor† for \( \delta_{d_e} \), which is followed by \( \delta_i \) and \( V_e \), while the rest of the parameters have relatively lower sensitivities. This fact implies that a partial factor significantly smaller than 1-0 need only be applied to \( N \), and the rest of partial factors should be kept nearly equal to 1-0.

The means of the sensitivity factors (Table 3, Figs 10(a) and (b)) for dead (permanent) load \( V_d \) and seismic load \( V_e \) suggest that they contribute to the loading side most of the time. However, it was found that, as the soil becomes stiffer and the eccentricity increases, for the footing cases with smaller dead loads, there is a tendency that \( V_d \) (and \( V_e \) for the cases when Meyerhof’s equation is used) contributes to the resistance side. The direct calculation of partial factors by the design value method is difficult in this case owing to the highly non-linear nature of the performance function: that is, \( i \), \( e \) and \( e \) are also functions of \( V_d \) and \( V_e \). It turned out that the partial factors obtained were unstable and not robust. (This problem was not encountered in the previous study, where the same methodology was applied to determine the partial factors for vertically loaded piles. In other words, the design point values obtained by FORM are divided directly by the corresponding nominal values of the basic variables to obtain partial factors. Note that the nominal values taken in this study are the mean values of the basic variables except for the seismic load, where the value corresponding to the 100-year return period is employed (assuming \( V_{e,100} \) is developed by \( S_{e,100} \)). The results are summarised in Tables 4(a) and 4(b), where the means and standard deviations (Std dev) of the partial factors obtained based on 216 cases are presented. All the failure probabilities have been obtained using only the mean values of the partial factors obtained for all the \( \phi \) cases.

The following observations are possible from the obtained results presented in Tables 4(a) and 4(b):

(a) For both Meyerhof’s and Vesci’s equations \( N \), has the mean largest rate of reduction of about 40% by the partial factor (0-63 and 0-66), followed by the seismic load, which is increased by 20% (i.e. the partial factor of 1-2). The other partial factors are kept at almost

† The sensitivity factor is defined as the normalised sensitivity of the performance function for a basic variable in normalised space (Melchers, 1999, p. 99).
unity except those concerning \( i \), where a 4% reduction of the nominal value is required.

(b) The partial factors for different friction angles \( \phi \) are also calculated. This is because the uncertainties associated with \( N_r \) for different \( \phi \) are significantly different, especially for Meyerhof’s equation, as indicated in Table 2. Both the biases and the standard deviations are larger for smaller \( \phi \). (This is not so for Vesic’s equation.) The calculated partial factors for \( N_r \) for Meyerhof’s equation are different for different \( \phi \) values, changing from 0.54 to 0.77. On the other hand, the change of the partial factors is less significant in the case of Vesic’s equation (0.65 to 0.69).

(c) The partial factors obtained for all cases in Tables 4(a) and 4(b) are applied to redesign footings, which are assessed using reliability analyses again. The results, unfortunately, did not exhibit any improvement as far as the introduction of a more uniform safety level in design for the 216 cases was concerned. This fact can be seen in Figs 11(a) and 11(b), where the probabilities of failure obtained by MCS for footings designed using the global safety factor (initial \( P_i \)) and the partial safety factors (final \( P_f \)) are compared.

It can be interpreted from the above that, owing to the dependence of the magnitudes of the biases and uncertainties on the friction angle \( \phi \) introduced in the model uncertainty, especially for Meyerhof’s equation, it is difficult to use a single partial factor for \( N_r \) to give a uniform safety level to the shallow foundation designs. In Figs 12(a) and 12(b), the probabilities of failure are obtained by the MCS for a large range of partial factors of \( N_r \) while keeping the partial factors for other parameters fixed (1.0 for \( e \) and \( V_o \), 0.96 for...
Table 5. Partial factors obtained for each case of $\phi$ through Monte Carlo simulations (MCS; one million simulations for each design case) and FORM using the partial factor determination curves (Fig. 12) for a target probability of failure of 0.200.

(a) Meyerhof (1963) equation

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<th>$\phi$</th>
<th>$\gamma_{r}$</th>
<th>$\gamma_{s}$</th>
<th>$\gamma_{e}$</th>
<th>$\gamma_{v}$</th>
<th>$\gamma_{i}$</th>
<th>Initial $P_f$</th>
<th>Final $P_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32°</td>
<td>Mean 0.96</td>
<td>0.62</td>
<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>0.129</td>
<td>0.200</td>
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<tr>
<td></td>
<td>(1.13)</td>
<td>(1.62)</td>
<td>(0.84)</td>
<td>(0.84)</td>
<td>(1.29)</td>
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<tr>
<td></td>
<td>Std dev COV</td>
<td>0.0242</td>
<td>0.0108</td>
<td>0.0523</td>
<td>0.0258</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35°</td>
<td>Mean 0.96</td>
<td>0.56</td>
<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>0.166</td>
<td>0.198</td>
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<tr>
<td></td>
<td>(0.97)</td>
<td>(1.49)</td>
<td>(0.85)</td>
<td>(1.34)</td>
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<tr>
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<td>Std dev COV</td>
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<td>0.0227</td>
<td>0.262</td>
<td>0.261</td>
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<tr>
<td>39°</td>
<td>Mean 0.96</td>
<td>0.47</td>
<td>1.00</td>
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<td>0.242</td>
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<td></td>
<td>(0.70)</td>
<td>(1.25)</td>
<td>(0.84)</td>
<td>(1.41)</td>
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<tr>
<td></td>
<td>Std dev COV</td>
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<td>0.0241</td>
<td>0.0702</td>
<td>0.0284</td>
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<tr>
<td>43°</td>
<td>Mean 0.96</td>
<td>0.36</td>
<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>0.240</td>
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<td></td>
<td>(0.75)</td>
<td>(1.23)</td>
<td>(0.84)</td>
<td>(1.39)</td>
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<td></td>
<td>Std dev COV</td>
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(b) Vesci (1975) equation

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<th>$\gamma_{s}$</th>
<th>$\gamma_{e}$</th>
<th>$\gamma_{v}$</th>
<th>$\gamma_{i}$</th>
<th>Initial $P_f$</th>
<th>Final $P_f$</th>
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<tbody>
<tr>
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<td>Mean 0.96</td>
<td>0.52</td>
<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>0.210</td>
<td>0.202</td>
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<td>(0.81)</td>
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<td>(1.24)</td>
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<td>Std dev COV</td>
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<tr>
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<td>(0.76)</td>
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<tr>
<td>43°</td>
<td>Mean 0.96</td>
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<td>(1.09)</td>
<td>(0.84)</td>
<td>(1.40)</td>
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<td>Std dev COV</td>
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<td>0.0656</td>
<td>0.0274</td>
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</table>

Numbers in parentheses are reliability indexes, $\beta$.

Initial $P_f$ = probability of failure calculated for designs using global factor of safety.

Final $P_f$ = probability of failure calculated for designs using partial factor of safety.

$i_r$ and 1-2 for $V_3$; referred to Table 4). One can choose different partial factors for different $\phi$ values to introduce a more uniform safety level in shallow foundation designs.

Our recommendation is to use the MCS results and set the target $P_f$ to 0.200 (equivalent to $\beta = 0.84$), which results in the partial factors $\gamma_N = 0.62$ for $\phi = 32^\circ$, $\gamma_N = 0.56$ for $\phi = 35^\circ$, $\gamma_N = 0.47$ for $\phi = 39^\circ$ and $\gamma_N = 0.36$ for $\phi = 43^\circ$ for Meyerhof’s equation, and $\gamma_N = 0.52$ for $\phi = 32^\circ$, $\gamma_N = 0.49$ for $\phi = 35^\circ$, $\gamma_N = 0.49$ for $\phi = 39^\circ$ and $\gamma_N = 0.44$ for $\phi = 43^\circ$ for Vesci’s equation respectively, with $\gamma_{r}, \gamma_{s}, \gamma_{v}$ and $\gamma_{i}$ kept constant at 0.96, 1.00, 1.00 and 1.20. It is apparent from the result that Vesci’s equation is more consistent in securing a uniform safety level for a wider range of $\phi$ values than Meyerhof’s equation.

The probabilities of failure obtained by MCS for footings designed using a global safety factor of 2.0 and for those using the partial factors recommended above (Figs 12(a) and 12(b)) are compared in Figs 13(a) and 13(b). The probabilities of failure calculated using the partial factors obtained are listed in Table 5. It can be seen that there is an improvement in the overall consistency of the failure probabilities in the designs using Meyerhof’s equation. It can also be seen that the designs using Vesci’s equation are more consistent for different soil friction angles than those using Meyerhof’s equation.

SUMMARY AND CONCLUSIONS

Reliability analysis carried out on the set of example cases based on a database of actual highway bridge footings showed that the 100-year failure probabilities obtained from the FORM analysis were generally lower than those from
the Monte Carlo simulations ($P_f$ of 0.019 by FORM compared with 0.027 by MCS for Meyerhof’s equation, and 0.086 compared with 0.207 for Brinch Hansen and Vesic’s equation). The reason for this discrepancy is thought to be the high non-linearity of the performance function and the approximation of the General Pareto distribution by the normal distribution in the FORM analysis.

The other important finding of this study was the dependence of the magnitude of the safety level on the friction angle $\phi$, especially when Meyerhof’s equation is used, which is due to the larger biases and uncertainties in the equation (see Table 2). The effect of this dependence was not so significant in Vesic’s equation, which gives a more uniform safety level to the design by using the same partial factor value for $N_c$ for different $\phi$.

It is the authors’ opinion that one has to be very careful in applying the approximation procedure of FORM analysis during the reliability analysis of a structure that has a highly non-linear performance function, such as a shallow foundation design with seismic loading. The authors consider the MCS to be a more accurate and straightforward tool than the FORM analysis.

The remaining problem in MCS is, however, that there is no established procedure, like the design value method, to determine the partial factor values. Such a method is highly desirable.

ACKNOWLEDGEMENTS

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REFERENCES


Honjo, Y. & Kusakabe, O. (2002). Proposal of a comprehensive foundation design code: Geo-code 21 ver.2. In Foundation...


