Why Consider Reliability Analysis for Geotechnical Limit State Design?

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**ABSTRACT:** There is a need to draw a clear distinction between accepting reliability analysis as a *necessary* theoretical basis for geotechnical design and downstream calibration of simplified multiple-factor design formats, with emphasis on the former. Reliability analysis provides a consistent method for propagation of uncertainties and a unifying framework for risk assessment across disciplines (structural and geotechnical design) and national boundaries. Simplified reliability-based design (RBD) equations are probably required for routine design at present, but their limitations have no bearing on the generality of reliability theory. If reliability analysis is accepted as the basis for developing multiple-factor formats, then it is necessary to define the characteristic values in an unambiguous way with reference to the probability distribution function. The key consideration is that the engineer should not be allowed to introduce additional conservatism into the design by using some lower bound value, when the RBD equations are calibrated using, say, mean parameters. The implementation of reliability-based LRFD equations, along with potential advantages and pitfalls, is discussed using a specific example. If the goal of LRFD is to maintain uniform reliability, the example shows that a single resistance factor is not adequate. In practice, it is probably sufficient to partition the parameter space (spanning typical ranges of deterministic and statistical parameters) into smaller domains and calibrate a single resistance factor for each domain. Deviations from the target reliability index can be controlled to an acceptable level by adjusting the sizes of the domains.

1  **INTRODUCTION**

Currently, the geotechnical community is mainly preoccupied with the transition from working or allowable stress design (WSD/ASD) to Load and Resistance Factor Design (LRFD). The term "LRFD" is used in a loose way to encompass methods that require all limit states to be checked using a specific multiple-factor format involving load and resistance factors. This term is used most widely in the United States and is equivalent to "Limit State Design (LSD)" in Canada. Both LRFD and LSD are philosophically akin to the partial factors approach commonly used in Europe, although a different
multiple-factor format involving factored soil parameters is used. The emphasis in LRFD or its equivalent in Canada and Europe is primarily on the re-distribution of the original global factor safety in WSD into separate load and resistance factors (or soil parameter partial factors).

The application of limit state design philosophy represents an important step towards more rational risk management in foundation engineering. Implementation of limit state design within a non-probabilistic framework, such as the empirical partial factors of safety method, does not appear to address adequately most of the serious drawbacks associated with the traditional factor of safety approach. For example, it is not clear how the empirical partial factors of safety method can promote communication, bridge structural and geotechnical design, assist in extrapolating the experience of safe practice to new conditions, or permit full advantage to be taken of improvements in the knowledge base. The adoption of such empirical methods might pave the way for gradual rationalization of the partial factors using probabilistic means, but the desirability of trading a known system for an unknown one solely on this basis is debatable.

Communication of risk within a transparent and rational framework is necessary in view of increasing interest in code harmonisation (Frank 2002, Honjo & Kusakabe 2002), public involvement in defining acceptable risk levels (Ellingwood 1999), and risk-sharing among client, consultant, insurer, and financier (Walker 1999). Activities in code harmonisation are particularly noteworthy. The advent of the World Trade Organisation (WTO) has added impetus to the formation of trading groups that result in multilateral free trade areas or bilateral free trade agreements. Traditionally, geotechnical engineering practice has always been viewed as a localised activity under the purview of the relevant federal and/or state authorities. However, the move towards greater economic cooperation and integration will require the elimination of some technical obstacles that exist as a consequence of differences in national codes and standards, and harmonisation of technical specifications. The “Agreement on Technical Barriers to Trade” issued by WTO can be found at http://www.wto.org/english/tratop_e/tbt_e/tbtagr_e.htm, of which Annex 3, Article F is of interest.

There is an urgent need to bridge structural and geotechnical design codes as well. Kulhawy & Phoon (2002) argued that geotechnical LRFD codes are fundamentally incompatible with reliability-based structural codes primarily because one or more of the following key elements are missing:

a. The primary objective in structural RBD is to achieve a minimum target reliability index across a specified domain of interest (e.g., foundation geometries and types, loading modes, soil conditions, etc.). Structural RBD requires deliberate and explicit choices to be made on the target reliability index, scope of calibration domains, and representative designs populating each domain. This is philosophically different from the objective of achieving designs comparable to WSD.

b. The secondary objective in structural RBD is to increase uniformity of reliability across the domain of interest, which is rarely emphasized and verified in geotechnical LRFD. In fact, the typical use of a single resistance factor for each loading mode is not adequate for this task.

c. Soil variability is the most significant source of uncertainty, but it is not quantified in a robust way (if at all) and incorporated explicitly in the code calibration process.

d. Probabilistic load models compatible with the relevant structural codes are not spelled out clearly. It is unclear if the original structural load models have been used for code calibration. Load combinations definitely are not amenable to simplified reliability analysis unless they are approximated as some lumped load parameters.

e. Rigorous reliability analysis using FORM is not used as the main tool to integrate loads, soil parameters, and calculation models in a realistic and self-consistent way, both physically and probabilistically.

f. No guidelines on selection of nominal or characteristic soil parameters are usually given. It is also unclear how resistance factors will be affected by the site conditions, measurement techniques, and correlation models used to derive the relevant design parameters.

The National Cooperative Highway Research Program project NCHRP 24-17 was initiated recently to provide: (a) recommended revisions to the driven pile and drilled shaft portions of section 10 of the AASHTO Specifications and (b) detailed procedures for calibrating deep foundation resistance factors (Paikowsky 2002). Hence, some of the above elements seem to be receiving some attention lately. In the
aftermath of recent natural hazards (e.g., Loma Prieta and Kobe earthquakes), the structural engineering profession currently is focusing on performance-based design aimed at meeting client-specific performance goals, in addition to complying with local building codes (Wen 2000, Buckle 2002). Efficient techniques for solving time-dependent reliability problems are needed for such problems. Clearly, theoretical developments in structural reliability and applications to probabilistic design are evolving rapidly. The gap between structural and geotechnical design appears to be widening.

There are strong practical reasons to consider geotechnical LRFD as a simplified reliability-based design procedure, rather than an exercise in rearranging the original global factor of safety. This calls for a willingness to accept the fundamental philosophy that: (a) absolute reliability is an unattainable goal in the presence of uncertainty and (b) probability theory can provide a formal framework for developing design criteria that would ensure that the probability of "failure" (used herein to refer to exceeding of any prescribed limit state) is acceptably small. In other words, geotechnical LRFD should be derived as the logical end-product of a philosophical shift in mindset to probabilistic design in the first instance and a simplification of rigorous reliability-based design into a familiar “look and feel” design format in the second. The need to draw a clear distinction between accepting reliability analysis as a necessary theoretical basis for geotechnical design and downstream calibration of simplified multiple-factor design formats, with emphasis on the former, is the main thesis of this paper.

The former provides a consistent method for propagation of uncertainties and a unifying framework for risk assessment across disciplines (structural and geotechnical design) and national boundaries. Other competing frameworks have been suggested (e.g., λ-method by Simpson et al. 1981, worst attainable value method by Bolton 1989, Taylor series method by Duncan 2000) but none has the theoretical breadth and power to handle complex real-world problems that may require nonlinear 3-D finite element or other numerical approaches for solution. In the development of Eurocode 7, much attention has been focused on the geotechnical aspects of code harmonisation (Frank 2002, Ovesen 2002, Orr 2002). This clearly takes precedence over safety aspects, but the time is perhaps ripe to decide if a theoretical platform is necessary to rationalise risk assessment. If the platform is not reliability analysis, then what alternative is available?

The need to derive simplified reliability-based design (RBD) equations perhaps is of practical importance to maintain continuity with past practice, but it is not necessary and increasingly fraught with difficulties when sufficiently complex problems are posed. The limitations faced by simplified RBD have no bearing on the generality of reliability theory. This is analogous to arguing that limitations in closed-form elastic solutions are related to elasto-plastic theory. The use of finite element software on relatively inexpensive and powerful PCs (with gigahertz processors, gigabyte of memory, and hundreds of gigabytes - verging on terabyte - of disk) permit real world problems to be simulated on an unprecedented realistic setting almost routinely. This paper presents some examples that would hopefully de-link some limitations of the simplified multiple-factor format from the underlying reliability framework. In doing so, the authors hope to encourage engineers to examine the more basic issue pertaining to the relevance of reliability theory in geotechnical design.

2 EVALUATION OF LRFD FORMATS USING RELIABILITY ANALYSIS

Much has been written concerning limitations inherent in the multiple-factor formats, usually with no reference to reliability theory (e.g., Baike 1985, Been 1989, Been et al. 1993, Valsangkar & Shriner 1991). An overview of these discussions has been given elsewhere (Kulhawy & Phoon 1996, Kulhawy & Phoon 2002, Phoon et al. 2003a). Some examples are given below to illustrate the usefulness of applying reliability theory to analyse limitations in the simplified design equations.

2.1 Characteristic Value

The definition of the characteristic or nominal value has been a major point of contention in geotechnical LRFD for many years (Green & Becker 2001). Simpson & Driscoll (1998) noted in their commentary to
Eurocode 7 that "their definition, in geotechnical terms, has been the most controversial topic in the whole process of drafting Eurocode 7". The authors believe that much of the resistance and controversy arose because there is misunderstanding as to the **scope of the definition**. As far as RBD is concerned, characteristic values must be definite only with reference to the probability distribution function (e.g., mean, mode, median, mean minus one standard deviation, fractile, etc.). This has been explained on numerous occasions (e.g., Been & Jefferies 1993, Dahlberg & Ronold 1993, Becker 1996a, Green & Becker 2001), but broad qualitative guidelines such as "cautious estimate of the value affecting the occurrence of the limit state" (CEN/TC250 1994) persist in the geotechnical LRFD literature.

The need to define the characteristic values in an unambiguous way with reference to the probability distribution function is clear within the context of reliability analysis as shown in Figure 1. It can be seen that the calibrated resistance and load factors are used to ensure consistent separation between the probability density functions describing the uncertain load and capacity. This can be demonstrated analytically by considering a well-known approximation to the lognormal reliability formula (Rosenblueth & Esteva 1972):

\[
\beta = \frac{\log_e \left( \frac{m_{FS}}{m_{F}} \right)}{\sqrt{\text{COV}_F^2 + \text{COV}_Q^2}}
\]

(1)

in which \(\beta\) = reliability index, \(m_{FS} = m_{Q}/m_{F}\) = mean factor of safety, \(m_Q\) and \(m_F\) = mean of capacity \((Q)\) and load \((F)\), \(\text{COV}_Q = s_Q/m_Q\) = coefficient of variation (COV) of capacity, \(\text{COV}_F = s_F/m_F\) = COV of load, and \(s_Q\) and \(s_F\) = standard deviation of capacity and load. To derive the LRFD format shown in Figure 1, the denominator in Equation 1 must be linearized as follows (Lind 1971):

\[
\sqrt{\text{COV}_F^2 + \text{COV}_Q^2} \approx 0.75 \left( \text{COV}_F + \text{COV}_Q \right)
\]

(2)

Substituting Equation 2 into 1 and rearranging the terms to fit the LRFD format shown in Figure 1, it can be shown that:

\[
\Psi = \frac{m_Q}{Q_n} \exp\left( -0.75 \beta \text{COV}_Q \right)
\]

(3a)

\[
\eta = \frac{m_F}{F_n} \exp\left( +0.75 \beta \text{COV}_F \right)
\]

(3b)

in which \(\Psi\) = resistance factor and \(\eta\) = load factor. As to be expected, reliability-calibrated resistance and load factors are functions of the target reliability index \((\beta)\) and the degree of uncertainty \((\text{COV}_Q\) and \(\text{COV}_F\)). However, it is equally clear from Equation 3 that efforts to rationalise resistance and load factors using reliability calibration to achieve a uniform level of acceptable risk will be undermined completely by the failure to adopt an unambiguous definition for Q_n and F_n with reference to their respective probability distribution functions.

The need to adopt an unambiguous probabilistic definition for characteristic values is often regarded as an unacceptable constraint on how the engineer should select representative design parameters from the usual physical point of view. An examination of reliability calibration as illustrated above will show that this is not the case. For example, an engineer familiar with a particular site may elect to use soil strength from a weak layer for some limit equilibrium analysis because he/she is confident that the failure surface passes mainly through this layer. If the characteristic value is defined at the mean, the engineer is required to estimate the mean strength of the layer of concern, but he/she is not required to disregard good geotechnical sense and be forced to estimate the average strength of the entire soil mass. The concern lies with the choice of the limit state function, not the definition of the characteristic strength!
There is no argument that an appreciation of the interaction between the geologic environment, loading characteristics, and geo-system response is of primary importance. It also is true that risk evaluation (or any other method of safety evaluation) is essentially of little consequence if the engineer were to assess soil properties incorrectly or to select an inappropriate failure mechanism for design. These qualifications are very important and have been highlighted on numerous occasions (Boden 1981, Semple 1981, Bolton 1983, Been 1989, Fleming 1989) but they should be taken as a given because they constituted part of our existing standards of good practice. A detailed discussion of these qualifications in the context of reliability analysis is given elsewhere (Phoon et al. 1993).

From a reliability perspective, the key consideration is that the engineer should not be allowed to introduce additional conservatism into the design by using, for example, some lower bound value, because the uncertainty in the design parameters already is built rationally into the RBD equations. Contrary to popular belief, the application of experience, sound judgment, and soil mechanics still is needed for all aspects of geotechnical design. Human intuition is not suited for reasoning with uncertainties and only this aspect has been removed from the purview of the engineer. Clearly, judgment is not undermined; instead, it is focused on those aspects for which it is most suited.

In principle, the definition of characteristic values is unrelated to reliability analysis. However, practical issues, such as simplicity, familiarity, and compatibility with the existing design approach, are important considerations that will determine if the simplified RBD design approach can gain ready acceptance among practicing foundation engineers (Phoon et al. 2003a). It is quite common for structural engineers to define characteristic strength as the 5% fractile. There is a tendency for geotechnical engineers to adopt the same concept (e.g., Becker 1996a), although its original rationale and applicability to geotechnical design are seldom analysed in the context of reliability theory as discussed below.

2.2 Baseline Technique

In structural RBD, it is common to represent variables with small sensitivity coefficients at their means and variables with large sensitivity coefficients at their fractiles. It is well known that doing so would allow a single set of partial factors to maintain a more uniform reliability over a range of uncertainties in the underlying random variables (e.g., Construction Industry Research & Information Association 1977). In other words, the partial factors are more robust compared to those derived using the mean of the variables. The simplest form is the baseline technique that involves the matching of a suitably chosen
fractile load and fractile capacity to achieve a consistent level of reliability without any partial factors as shown below:

\[ Q_{\omega} = F_{50} \]  

(4)  

in which \( Q_{\omega} = \omega \% \) fractile capacity and \( F_{50} = 98 \% \) fractile load or 50-year return period load. Note that a T-year return period load is equal to \( 100(1 - 1/T)\% \) fractile load. In this example, the capacity \( Q \) is modeled as a lognormal random variable with a COV between 10 and 20\%. Hence, \( Q_{\omega} \) can be related to the mean and coefficient of variation of \( Q \) \((m_Q \text{ and } \text{COV}_Q)\) using the following expression:

\[ Q_{\omega} = m_Q \exp[\Phi^{-1}(\omega/100)\sqrt{\ln(1+\text{COV}_Q^2)} - 0.5\ln(1+\text{COV}_Q^2)] \]  

(5)  

in which \( \Phi^{-1}(\cdot) \) = inverse standard normal cumulative function. The definition of a 5\% fractile is illustrated in Figure 2a. The weather-related load effect \( F \) is assumed to be proportional to the square of the wind speed as follows:

\[ F = kV^2 \]  

(6)
Figure 3  Probability of failure implied by baseline design equation: $Q_\omega = F_{50}$.

in which $V = \text{Gumbel random variable with a COV of 30\% and } k = \text{deterministic constant}$. The 50-year return period load ($F_{50}$) is given by $k v_{50}$, in which $v_{50}$ is the 50-year return wind speed shown in Figure 2b. For a Gumbel probability distribution, $v_{50}$ can be evaluated in terms of the mean and coefficient of variation of $V$ ($m_V$ and $\text{COV}_V$) using the following expression:

$$v_{50} = m_V (1 + 2.59 \times \text{COV}_V)$$  \hspace{1cm} (7)

The baseline reliability calibration procedure involves adjusting the fractile ($\omega$) in Equation 4 until a target annual probability of failure is achieved. To calculate the probability of failure implied by Equation 4, the First-Order Reliability Method (FORM) is used. Results of the reliability calculations using FORM for a range of fractiles (or exclusion limits) and capacity coefficients of variation (COVs) are shown in Figure 3. It is clear that a reasonably consistent target probability of failure of 1\% can be achieved by simply matching the 98\% fractile load ($F_{98}$) with the 5\% fractile capacity ($Q_5$). No partial factor is needed in Equation 4. In particular, the probability of failure is fairly insensitive to changes in the capacity COV from 10 to 20\% and capacity fractile from 5 to 10\% (shaded area in Figure 3). If the 50-year return period load in Equation 4 is replaced by some higher return period load, the probability of failure would be simply reduced by a factor of $T/50$, in which $T$ is the return period in years. However, this simple empirical reduction factor only is applicable to cases where the COV of the capacity in Equation 4 is less than 25\% (Peyrot & Dagher 1984).

For geotechnical design, the above advantages cannot be realised because COVs for geotechnical parameters are much higher than those for structural strengths. The typical ranges of COVs for geotechnical parameters also are much wider. Coefficients of variation ranging from 30 to 70\% are not unrealistic for common geotechnical parameters (Phoon & Kulhawy 1999a, 1999b). If a lower target probability of failure of 0.25\% is desired (reliability index = 2.81), a 1\% capacity fractile should be used when the capacity COV is 30\% (Figure 3). If this 1\% capacity fractile is retained in Equation 4 when the
capacity COV changes to 50%, a partial factor of $Q_{2.6}/Q_{1} = 1.20$ (from Equation 5) must be applied. The original robustness of Equation 4 is lost.

2.3 LRFD Technique

The reliability-calibrated resistance and load factors given by Equation 3 include the target reliability index ($\beta$) and the underlying uncertainties (COV Q and COV F) rationally into the design process. The implementation of these equations, along with potential advantages and pitfalls, is discussed using a specific example for illustration.

The example considered is shown in Figure 4. The ultimate lateral capacity ($H_u$) of a free-head rigid pile in homogeneous sand can be evaluated using Brom’s simplified yield stress distribution:

$$H_u = 0.5 \frac{\gamma BD^3 Kp}{(e + D)} = 0.5 \frac{\gamma BD^3 \tan^2 \left(45^\circ + \phi/2 \right)}{(e + D)} \tag{8}$$

in which $B$, $D =$ diameter and length of pile, $e =$ eccentricity, $\gamma =$ effective unit weight of sand, $\phi =$ effective stress friction angle, and $Kp =$ Rankine coefficient of passive soil stress.

Two main sources of geotechnical uncertainties can be distinguished. The first arises from the evaluation of design soil properties, such as the effective stress friction angle. The second source arises from geotechnical calculation models. Although many geotechnical calculation models such as Equation 8 are “simple”, reasonable predictions of fairly complex soil-structure interaction behavior still can be achieved through empirical calibrations. Because of our geotechnical heritage that is steeped in such empiricisms, model uncertainties can be significant. Even a simple estimate of the average model bias is crucial for reliability-based design. If the model is conservative, it is obvious that the probabilities of failure calculated subsequently will be biased, because those design situations that belong to the safe domain will be assigned incorrectly to the failure domain, as a result of the built-in conservatism.

![Figure 4. Simplified Broms approach for laterally loaded rigid pile in sand (Broms 1964).](image)
Based on a detailed study of 55 model-scale laboratory tests and 22 full-scale field tests, Phoon & Kulhawy (2003) proposed that $H_u$ computed using Broms simplified approach could be corrected by a model factor as follows:

$$H_h = M H_u$$

(9)

in which $H_h =$ measured hyperbolic capacity and $M$ is a lognormal random variable with mean 1.3 and coefficient of variation (COVM) about 0.4 (Figure 5). It can be seen that the reliability framework permits full advantage to be taken of improvements in the knowledge base by updating $M$ when more load tests are available.

The basic objective of reliability-based design (RBD) is to ensure that the probability of failure of a component does not exceed an acceptable threshold level. While the above objective is satisfied if the probability of failure of a component lies far below the threshold, it is clear that the design is not economical. Therefore, a realistic interpretation of the design objective would include the implicit requirement that the probability of failure does not depart significantly from the threshold (see item b in Section 1 – Introduction).

For the simple design problem shown in Figure 4, the reliability-based design objective can be formally stated as follows:

$$p_f = \text{Prob}(H_h < F) = \text{Prob}(G < 0) \leq p_T$$

(10)

in which Prob($\cdot$) = probability of an event, $p_f =$ probability of failure, $G =$ performance or limit state function $= H_h - F$, and $p_T =$ acceptable target probability of failure. Reliability-based design, as exemplified by Equation 10, clearly is more rational than the factor of safety approach because the design risk is evaluated explicitly using probability theory. The engineer is able to make a conscious choice on an acceptable level of design risk and then proceed to a set of design dimensions that are consistent with the above choice. Logical consistency between the computed design risk and the uncertainties inherent in the design process is assured by reliability analyses, such as the First-Order Reliability Method.

Reliability-based design in the form of Equation 10 involves the repeated use of reliability routines, such as FORM, to evaluate the probabilities of failure of trial designs until the computed probability of failure is reasonably close to the chosen threshold level. At present, this is not done for routine designs because reliability analyses are perceived as difficult to perform (Whitman 1984). Hence, simplified LRFD equations such as Equation 3 are developed to serve as a bridge between reliability analyses and current practice (factor of safety). This simplification step can introduce some difficulties as discussed below. Nevertheless, it must be clearly recognised that these difficulties faced by simplified design equations have no bearing on the generality of reliability theory.
The performance of the laterally-loaded pile shown in Figure 4 cannot be assessed with absolute certainty because of uncertainties in the capacity ($H_h$) and load ($F$). It is tempting to assume that the performance function, $G = H_h - F$, is a linear function of two independent normal random variables ($H_h$ and $F$). By doing so, it is possible to compute the reliability index using a convenient closed-form solution (Equation A.2 in Appendix). However, it is clear from Equations 8 and 9 that the uncertainties originate from more basic random variables such as $\phi$ and $M$, and the performance function ($G$) is actually nonlinear when expressed in terms of these basic variables. It is possible to linearise $G$ using Taylor series and proceed with reliability computation using Equation A.2. This is known as the First-Order Second-Moment (FOSM) method. Such an approach is overly-simplified because FOSM is known to suffer from the problem of invariance. The simple example given in the Appendix is sufficient to demonstrate this important theoretical flaw. Actual performance functions are much more complicated and it is difficult to assess the accuracy of FOSM without comparison with more accurate methods such as FORM or simulation. It is best to avoid FOSM in view of its theoretical flaw.

A more common simplified reliability basis for developing geotechnical LRFD codes is the lumped parameter lognormal formula given by Equation 1 (e.g., Barker et al. 1991, Becker 1996b, Yoon & O'Neil 1997, Paikowsky 2002). This technique would be evaluated rigorously using FORM and simulation. To conduct this study, the following assumptions are made:

a. The load ($F$) is lognormally distributed with $COV_F$ between 10 and 20% (within the range of 7% and 30% noted by Becker 1996b). The mean load ($m_F$) is typically 0.7 to 1.0 of the nominal load ($F_n$) (Becker 1996b). An average value of $m_F/F_n = 0.85$ is chosen.

b. The nominal capacity ($Q_n$) is calculated from Equation 8 with a realistic estimate of $\phi$ (interpreted as equal to mean of $\phi$ from hereon).

c. The model factor ($M$) is lognormally distributed with mean 1.3 and $COV_M = 0.4$.

d. The effective stress friction angle ($\phi_e$) is normally distributed with mean between 35° and 45°, and $COV_{\phi_e}$ between 5% and 20% (Phoon & Kulhawy 1999a).

e. The effective unit weight of sand ($\gamma$) and geometric dimensions ($B, D, e$) are deterministic.

Under these assumptions, the resistance and load factors for this example can be computed from Equation 3 as:

$$\Psi = 1.3 \exp (-0.75 \beta \ COV_{Hh})$$

$$\eta = 0.85 \exp (0.75 \beta \ COV_F)$$

(11a)

(11b)

One difficulty of the lumped parameter approach that is apparent from this example is the propagation of uncertainties from basic inputs ($COV_{\phi}$ and $COV_M$) to the desired $COV$ of lumped capacity ($COV_{Hh}$). One common approach is to estimate $COV_{Hh}$ using the first-order second-moment method:

$$COV_{Hh} \approx \sqrt{COV_{M}^2 + COV_{Kp}^2}$$

(12)

in which $COV_{Kp} = COV$ of Rankine coefficient of passive soil stress ($K_p$). However, $K_p$ is a strongly nonlinear function of $\phi$. An attempt to estimate $COV_{Kp}$ using the first-order second-moment method again would result in the following equation:

$$COV_{Kp} \approx \frac{\pi \phi \ COV_{\phi}}{90^\circ \ cos \ \phi}$$

(13)

in which $\phi$ is expressed in degrees. A comparison between Equation 13 and simulation is shown in Figure 6. It is clear that the first-order second-moment method only works well up to $COV_{\phi} \leq 10\%$. This is an example where the first-order second-moment method does not work well even for relatively
small COVs (say less than 20%) because of strong functional nonlinearity. However, simulation results for the case of mean $\phi = 40^\circ$ and $\text{COV}_{\phi} = 10\%$ seem to indicate that a lognormal distribution for the lumped capacity parameter ($H_h \propto M K_p$) is adequate in this example (Figure 7). Besides simulation, there is no simple method for verifying approximate lognormality in the capacity. There are certainly no convenient theoretical results to prove that $H_h$ is lognormal if $\phi$ is normal and $M$ is lognormal (see Equations 8 and 9). In fact, a closer scrutiny using the normal probability plot (Figure 8) reveals that the normality of $\log_e(MK_p)$, and hence lognormality of $MK_p$, is rejected at the customary 5\% level of significance, despite the linearity of the plot. The First-Order Reliability Method works directly with the input distributions for $\phi$ and $M$ – there is no necessity to determine the distribution of $H_h$. It also provides more accurate solutions as shown in Table 1.

To examine the variation of $\Psi$ and $\eta$ given by Equation 11, $\text{COV}_{H_h}$ is evaluated approximately from Equation 12 assuming that $\text{COV}_{K_p}$ is roughly twice $\text{COV}_{\phi}$ (Figure 6):

$$\text{COV}_{H_h} \approx \sqrt{\text{COV}_{MK}^2 + 4 \text{COV}_\phi^2}$$  \hspace{1cm} (14)$$

The relationships between $\beta$, $FS = Q_n/F_n$, and $\Psi$ are shown in Figure 9 for typical values of $\text{COV}_F$ and $\text{COV}_\phi$. The load factor $\eta$ can be deduced from Figure 1 as $\Psi \times FS$. Note that different $FS$ (or $\Psi$ and $\eta$) are needed to achieve a consistent level of risk (constant $\beta$) for different degrees of uncertainties in load and effective stress friction angle. For example, to maintain a target $\beta = 3.5$, it is necessary to increase $FS$ from 2.5 to 4.9 or decrease $\Psi$ from 0.44 to 0.29, when $\text{COV}_F$ and $\text{COV}_\phi$ increase. If the goal of LRFD is to maintain uniform reliability, then it is evident that a single resistance factor is not adequate. The COVs considered vary over a narrow range and their magnitudes are comparable to typical values appearing in structural RBD. Nevertheless, the reliability index is sensitive to small changes in COV in this example because the design parameters are very influential. Hence, it is not safe to extrapolate experiences from structural RBD (e.g., Figure 3) without performing a systematic reliability study. In practice, it is probably not necessary to determine resistance factors to the level of precision shown in Figure 9. The proposal originating from structural RBD (e.g., Ellingwood et al. 1980) and implemented rigorously for foundation RBD by Phoon et al. (1995) is to partition the parameter space (spanning typical ranges of deterministic and statistical parameters) into smaller domains and calibrate a single resistance factor for each domain. Deviations from the target reliability index are expected, but these deviations can be controlled to an acceptable level by adjusting the sizes of the domains. Phoon et
al. (2003b) illustrated application of this sub-domain method for simplified reliability-based design of spread foundations in uplift.

![Graphs showing simulation inputs and outputs](image)

Figure 7. Simulation inputs are (a) normal $\phi$ and (b) lognormal $M$; output are (c) $K_p$ and (d) $H_h \propto M K_p$.

Table 1. Comparison between closed-form lognormal formula, FORM, simulation for reliability computations.

<table>
<thead>
<tr>
<th>COV $\phi$</th>
<th>COV $F$</th>
<th>COV $\ln \beta$</th>
<th>Target $\beta$</th>
<th>$FS^b$</th>
<th>Actual $\beta$ $FS^b$</th>
<th>Lognormal $c$</th>
<th>FORM $d$</th>
<th>Simulation $e$</th>
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- $FS = Q/F_n = \eta/\Psi$ ($\eta$ & $\Psi$ from Equation 3)
- Exact version of Equation 1: $\beta = \log e [m_{FS} \sqrt{(1 + COV_{F}^2)/(1 + COV_{\phi}^2)}] / [\sqrt{\log e [(1 + COV_{F}^2)/(1 + COV_{\phi}^2)]}]$
- Based on Solver from EXCEL™ (Low & Phoon 2002)
- Based on 20,000 random samples from EXCEL™
Figure 8. Normal probability plot for $\log_e(MK_p)$ from MINITAB™.

Figure 9. Relationships between $\beta$, FS, and $\Psi$ for laterally loaded pile example.
CONCLUSIONS

This paper advocates that geotechnical LRFD should be derived as the logical end-product of a philosophical shift in mindset to probabilistic design in the first instance and a simplification of rigorous reliability-based design into a familiar “look and feel” design format in the second. There is a need to draw a clear distinction between accepting reliability analysis as a necessary theoretical basis for geotechnical design and downstream calibration of simplified multiple-factor design formats, with emphasis on the former. Reliability analysis provides a consistent method for propagation of uncertainties and a unifying framework for risk assessment across disciplines (structural and geotechnical design) and national boundaries. There is a practical need to develop simplified reliability-based design (RBD) equations for routine design at present, but it may not be appropriate for sufficiently complex problems. Regardless, the limitations faced by simplified RBD have no bearing on the generality of reliability theory.

If reliability analysis is accepted as the basis for developing multiple-factor formats, then it is necessary to define the characteristic values in an unambiguous way with reference to the probability distribution function. The key consideration is that the engineer should not be allowed to introduce additional conservatism into the design by using some lower bound value, when the RBD equations are calibrated using, say, mean parameters. It is true that risk evaluation (or any other method of safety evaluation) is essentially of little consequence if the engineer were to assess soil properties incorrectly or to select an inappropriate failure mechanism for design. These qualifications are very important, but they should be taken as a given because they constituted part of our existing standards of good practice.

The implementation of reliability-based LRFD equations, along with potential advantages and pitfalls, is discussed using a specific example for illustration. If the goal of LRFD is to maintain uniform reliability, the example shows that a single resistance factor is not adequate. In practice, it is probably not necessary to swing to the other extreme of providing continuously varying resistance factors. The proposal originating from structural RBD is to partition the parameter space (spanning typical ranges of deterministic and statistical parameters) into smaller domains and calibrate a single resistance factor for each domain. Deviations from the target reliability index can be controlled to an acceptable level by adjusting the sizes of the domains.

APPENDIX

Problem of invariance for FOSM (First-Order Second-Moment) Method

Consider the classic reliability problem with one capacity (Q) and one load (F) random variable that are independent and follow a bivariate normal distribution. The performance function \(G_1\) and the reliability index \(\beta_1\) are given by:

\[
G_1 = Q - F
\]

\[
\beta_1 = \frac{m_Q - m_F}{\sqrt{s_Q^2 + s_F^2}}
\]

Equation A.2 is a closed-form solution. Another \textit{mechanically equivalent} performance function can be stated for this problem:

\[
G_2 = (Q - F)^3
\]
It can be seen that \((G_2 < 0)\) produces exactly the same failure domain as \((G_1 < 0)\) and one must expect the reliability for \(G_2\) to be the same. Nevertheless, this is not the case if FOSM is used. The key problem lies with linearization of \(G_2\) about the mean values of \(Q\) and \(F\) using Taylor series expansion:

\[
G_{2L} = \left( m_Q - m_F \right)^3 + 3\left( m_Q - m_F \right)^2 \left( Q - m_Q \right) - 3\left( m_Q - m_F \right)^2 \left( F - m_F \right)
\]

(A.4)

in which \(G_{2L}\) = linearised version of \(G_2\). The mean and standard deviation of \(G_{2L}\) are respectively:

\[
m_{GL} = \left( m_Q - m_F \right)^3
\]

\[
s_{GL} = 3\left( m_Q - m_F \right)^2 \sqrt{s_Q^2 + s_F^2}
\]

(A.5)

This results in a new reliability index that is only one-third of \(\beta_1\) (the correct solution):

\[
\beta_2 = \frac{m_{GL}}{s_{GL}} = \frac{m_Q - m_F}{3\sqrt{s_Q^2 + s_F^2}} = \frac{\beta_1}{3}
\]

(A.6)

Although this example is very simple, it clearly illustrates the severe invariance problem of FOSM with minimum mathematical details. This problem is widely known in the structural reliability community and has been solved by Hasofer and Lind (1974), when they proved mathematically that an invariant measure of reliability can only be obtained by considering the nearest distance of the limit state function from the origin of a standard Gaussian space.

REFERENCES


