Benefit Evaluation of the Recreation Facility Construction by Using Fuzzy Utility

Shinichi MUTO*, Akiyoshi TAKAGI**

* Department of Civil Engineering and Urban Design, Osaka Institute of Technology,
5-16-1 Ohmiya Asahi-ku, Osaka 535-8585, JAPAN, muto@civil.oit.ac.jp
** Department of Civil Engineering, Gifu University,
1-1, Yanagido, Gifu 501-1193, JAPAN, a_takagi@cc.gifu-u.ac.jp

***Abstract*— When we evaluate the construction of infrastructure by the economic flame, we have two important assumptions, the perfect information and the rational behavior. But we understand there is always fuzziness in those. In this paper, we built the benefit evaluation model by using the fuzzy utility, in order to consider the fuzziness in decision-making of economic agents on their activities. We introduced the fuzziness for the utility maximizing behavior on the basis of the economic theory, and showed the benefits measured as fuzzy number. Ｗe carried out the simulation analysis for the supposed recreation facility construction. Though we set the 10% fuzziness, we cleared the benefit had the large width.**

***Keywords*— Benefit evaluation, Consumer behavior model, Fuzzy utility function**

I. INTRODUCTION

It has been developed benefit evaluation methods based on the economic theory in order to measure the effectiveness of public facility construction [1], [2]. In these methods, the concept of utility plays very important role. The reason is that benefits are defined by the utility difference for the facility constructing or not.

It has been premised in the past researches, a utility is determined uniquely if an economic situation is limited. When a utility is set uniquely, the commodity consuming behavior of a person is also determined uniquely. However, his behaviors are not completely the same even if the economic situations are the same. Example, recreation behaviors are changed by the mental state or the day's weather. So, peoples' behaviors have to be grasped not to be decided uniquely.

This matter is on the variation within the same person, but is not on the variation among people. The latter is defined as the randomness that is the error of observation because there are many various persons and activities in a society, and that can be represented by the random utility theory [3]. On the other hand, the former is defined as the fuzziness that is within the same person, and that can not be represented by the random utility theory. In this study, we consider that his behaviors change because of having fuzziness in the value criterion of a person.

In this paper, we propose the benefit evaluation method considered the fuzziness in peoples' behaviors by using the concept of fuzzy utility. Though the framework of benefit evaluation is the same as the past model, the utility function used in our model is extended to fuzzy model. And we apply this model to the actual recreation facility construction and confirm its availability.

II. FORMULATION OF THEORETICAL MODEL

A. Household (Outline)

In this model, we focus on the behaviors consuming recreation service of household. Because these are not determined by only the economic principles at all, we can not deal with definitely the demand of visiting the recreation facility, in general. That is, those behaviors have some fuzziness.

So we modeled the behaviors consuming recreation service by using the fuzzy utility maximization programming. And the benefits of the recreation facilities improvement are defined by the fuzzy utility level solved from our model.

The utility function of household is shown generally like this [4].

$$u = u(z_H, u_R, s_H) \tag{1}$$

Where, z_H : composite goods consumption, u_R : recreation service, s_H : leisure time, u : direct utility function.

We try to extend this utility function to the fuzzy one $\tilde{u} = [u^-, u, u^+]$. We specified it as follows by fuzzy coefficient $\tilde{A}$, and its intuitional figure is shown in Figure 1. In there, we present the concept of fuzzy utility focusing on only z_H and u_R .

$$\tilde{u} = \tilde{u}(\tilde{A}, z_H, u_R, s_H) \tag{2}$$

Where, $\tilde{A}$: fuzzy coefficient $[A^-, A, A^+]$.

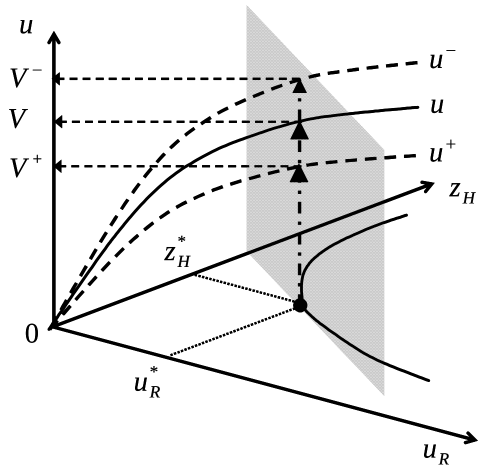

Figure 1. Intuitional figure of the fuzzy utility function

The utility maximizing behavior is formulated by the fuzzy utility function as below.

$$\max \quad \tilde{u} = \tilde{u}(\tilde{A}, z_H, u_R, s_H) \tag{3a}$$

$$\text{s.t.} \quad p_M z_H + c_R u_R + w s_H = wT_H + \pi_M + \pi_R - \tau_H [\equiv \Omega] \tag{3b}$$

Where, T_H : total available time, p_M : composite goods price, c_R : unit cost of recreation service, w : wage, π_M : distribution income from composite goods firm, π_R : distribution income from recreation firm, τ_H : lump sum tax, Ω : full income.

By solving the optimal programming in (3), we obtain demand functions as z_H , u_R , s_H , respectively. And the substitution of those demand functions into the objective function gives an indirect utility function as fuzzy number.

$$\tilde{V} = \tilde{V}(\tilde{A}, p_M, c_R, w, \Omega) \tag{4}$$

The (4) indicates that the utility level defined for voluntary commodity consumption has a width. This idea is corresponded with the case fixing the indirect utility V_0 in figure 2. That case gives three indifference curves (figure 2). In other words, an indifference curve has also a width like as figure 3. That fact implies the consumer is possible to change his consumption level even if his utility level is the same.

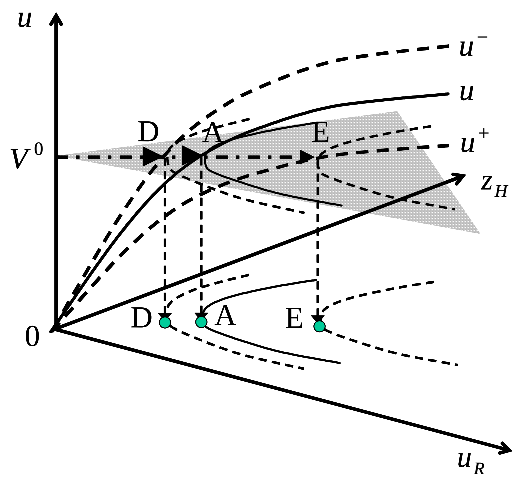

Figure 2. Fuzzy indifference curves

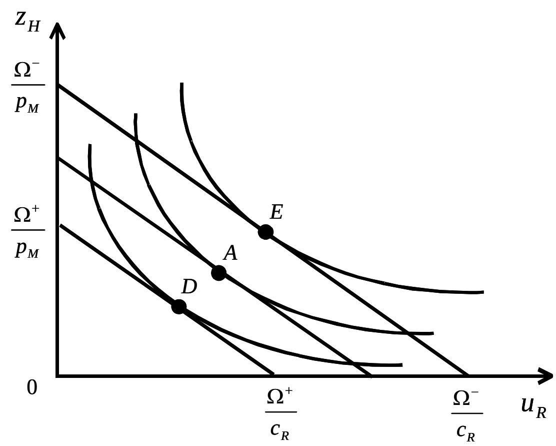

Figure 3. Fuzzy indifference curves and budget constraint line

B. Household (Recreation service consuming behavior)

We try to specify the indirect utility $\tilde{V}$. Here, we focus on the recreation service consuming behavior, so the composite goods price and wage are supposed as fix. And fuzzy coefficient is assumed to influence for the term of recreation service unit cost c_R. The $\tilde{V}$ is formulated with the linear type for the full income as below.

$$\tilde{V} = -\xi \cdot \tilde{A} \frac{\alpha_1}{\alpha_2} \exp(\alpha_2 c_R) + \mu \Omega \tag{5}$$

Where, ξ, α_1, α_2 : parameters.

By using the Roy's identify, the recreation demand function is obtained. The one becomes also fuzzy number.

$$\tilde{u}_R = \tilde{A} \cdot \alpha_1 \exp(\alpha_2 c_R) \tag{6}$$

C. Household (Recreation service producing behavior)

The supply of recreation service is formulated by the concept of household production [5]. Household produces the recreation service by inputting the recreation goods and access times to recreation facility. We formulate this behavior by the product cost minimizing program with the product technology of Cobb-Douglas type.

$$c_R u_R = \min_{z_R, t_R} [p_R z_R + w t_R] \tag{7a}$$

$$\text{s.t.} \quad u_R = \frac{1}{\eta_H(r)} z_R^{\alpha^s} t_R^{\beta^s} \tag{7b}$$

Where, z_R : recreation goods inputting volume, t_R : access times for recreation, η_H : product technical parameter, α^S, β^S : parameters..

The r is the recreation facility constructing level. Here, we interpret that the recreation facility improvement project revealed by the change of the product technical parameter η_H. The (7) yields to recreation goods and recreation access time's demand functions. Substituting them into (7), we obtain the unit cost of recreation service.

$$c_R = \eta_H(r) \left[p_R \left[\frac{\alpha^s w}{\beta^s p_R} \right]^{\beta^s} + w \left[\frac{\beta^s p_R}{\alpha^s w} \right]^{\alpha^s} \right] \tag{8}$$

The recreation goods price also is assumed to fix. The second term of (8) is considered as fix. We replace it with q. The q is what you called generalized cost. So the c_R is represented like this.

$$c_R = \eta_H(r) \cdot q \tag{9}$$

Here, the $\eta_H(r)$ is specified as follows.

$$\eta_H(r) = 1 + \frac{\alpha_3 r_K}{\alpha_2 q} + \frac{\alpha_4 r_S}{\alpha_2 q} \tag{10}$$

Where, r_K : constructing dummy of riverside park (with 1. without 0), r_S : constructing dummy of amenity riverside park (with 1. without 0).

Substitution of (10) into (9) gives c_R as below.

$$c_R = q + \frac{\alpha_3}{\alpha_2} r_K + \frac{\alpha_4}{\alpha_2} r_S \tag{11}$$

And Substituting (11) into (5) and (6), the fuzzy utility level and demand function become like this.

$$\tilde{V} = -\mu \cdot \tilde{A} \frac{\alpha_1}{\alpha_2} \exp[\alpha_2 q + \alpha_3 r_K + \alpha_4 r_S] + \mu \Omega \tag{12a}$$

$$\tilde{u}_R = \tilde{A} \cdot \alpha_1 \exp[\alpha_2 q + \alpha_3 r_K + \alpha_4 r_S] \tag{12b}$$

III. Definition of Benefit

Here, the benefits of the riverside park or the amenity riverside park construction are defined. We express the constructing projects as change of constructing dummy r_K, r_S. From this result, a household fuzzy utility level is expected to raise $\tilde{V}^A \rightarrow \tilde{V}^B$ (A, B : without project and with, respectively). The benefits are defined for the difference of these utility levels by using the concept of equivalent variation (EV) as follows [6].

$$\tilde{V}\left(q^A, r^A, \Omega^A + EV\right) = \tilde{V}^B \tag{13}$$

The (13) implies that the benefits are measured by the amount of compensation for the state without the project under the condition equaling the both utility levels of with project and without. In our model, the utility level has been gotten as fuzzy number. Here, we have to search the amount of compensation to equal both fuzzy utility levels. We apply the necessity index to judge the correspondence of fuzzy utility levels [7].

The equation index using the necessity measurement is yielded as below.

$$Nes\left(\tilde{V}^A = \tilde{V}^B\right) = \min\left[Nes\left(\tilde{V}^A \subseteq \tilde{V}^B\right), Nes\left(\tilde{V}^A \supseteq \tilde{V}^B\right) \right] \quad (14)$$

where,

$$Nes\left(\tilde{V}^A \subseteq \tilde{V}^B\right) = \inf\ \max\left[1-\mu_{V^A}, \mu_{V^B}\right] \quad (15a)$$

$$Nes\left(\tilde{V}^A \supseteq \tilde{V}^B\right) = \inf\ \max\left[\mu_{V^A}, 1-\mu_{V^B}\right] \quad (15b)$$

with, $\tilde{V}^A = \tilde{V}\left(q^A, r^A, \Omega^A + EV\right)$, μ : value of membership function.

The (15a) shows the degree of necessity to be included $\tilde{V}^A$ in $\tilde{V}^B$, and the (15b) does the one for $\tilde{V}^A$ to include $\tilde{V}^B$. And we define the index of correspondence by considering that $\tilde{V}^A = \tilde{V}^B$ is regarded as $\tilde{V}^A \subseteq \tilde{V}^B$ and $\tilde{V}^A \supseteq \tilde{V}^B$ as like (14).

The possibility measurement is generally used as the index of correspondence. But this index only indicates the possibility of correspondence. Because of being the fear that the benefit becomes to excessive evaluation by applying the possibility measurement, we do not accept the possibility measurement but the necessity measurement.

The concrete method to calculate the necessity measure is shown in figure 4 for the case of (15a).

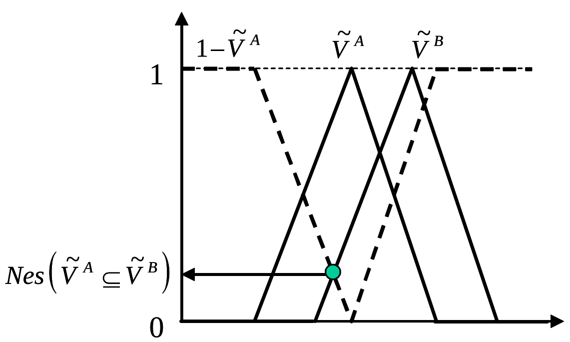

Figure 4. Calculation of necessity measurement

While the $Nes\left(\tilde{V}^A = \tilde{V}^B\right)$ has some value, we interpret that $\tilde{V}^A$ corresponds with $\tilde{V}^B$. The degree of correspondence is obtained by the value of $Nes\left(\tilde{V}^A = \tilde{V}^B\right)$. However, the upper limit of $Nes\left(\tilde{V}^A = \tilde{V}^B\right)$ is 0.5.

IV. Benefit Evaluation of Riverside Parks

A. *Outline of projects*

We apply our model to actual riverside park and amenity river park constructions in the Yamazaki River at Nagoya City. The riverside park has scenery shore or walking road and so on. The amenity river park is improved the water quality of river added on the riverside park facilities. The project cost is about 4 billion yen for the riverside park construction, and about 16 billion yen for the amenity river park.

Here, we utilize the opinion survey examined for the residents living around the Yamazaki River in 1986. Though the gotten data grow old, we try to confirm the impact applying the fuzzy utility function for the actual projects.

B. *Result of parameter estimations*

The parameters of fuzzy utility function are estimated from data of the opinion survey. That survey was examined for the head of household and dependent family. So we made out an indirect utility function and demand function of dependent family. But their forms are the same as the one of household in (12).

$$\tilde{V} = -\mu \cdot \tilde{A}\frac{\beta_1}{\beta_2}\exp\left[\beta_2\, q_D + \beta_3\, r_K + \beta_4\, r_S + \beta_5 A\delta\right] + \mu\Omega_D \quad (16a)$$

$$\tilde{u}_R = \tilde{A}\cdot\beta_1 \exp\left[\beta_2\, q_D + \beta_3\, r_K + \beta_4\, r_S + \beta_5 A\delta\right] \quad (16b)$$

Where, q_D : generalized cost of visiting the park of dependent family, A : age, δ : age dummy (over fifteen yeas old: 1, under fifteen yeas old: 0), Ω_D : full income of dependent family.

The generalized cost is calculated as walking speed 65 [m/min] and time value of household 1,000 [yen/hour] or dependent family 468[yen/hour].

We estimated parameters of the utility function by taking the logarithm for the demand functions of (12b) and (16b) and using linear regression,. Its results are shown in table 1. In table 1, the values of t-statistics of α_1, β_1 are shown to the logarithm for the parameters as

Table 1. Result of parameter estimation

	Generalized cost	Riverside park	Amenity river park	Age	Coefficient of correlation
The head of household	α_1	α_2	α_3	α_4		R
	1.9389 (15.94)	-4.47×10^{-3} (-4.731)	1.2087 (9.154)	1.3528 (10.01)		0.906
Dependent family	β_1	β_2	β_3	β_4	β_5	R
	4.5594 (22.08)	-1.60×10^{-2} (-12.34)	0.9567 (7.233)	1.2283 (9.287)	-7.88×10^{-3} (-2.091)	0.910

( ) : t-statistics

Table 2. Result of benefit evaluation for the riverside park

Distance until park		100m	300m	500m	700m	1000m
The head of household	Min	0.93 (0.155)	0.74 (0.124)	0.59 (0.098)	0.47 (0.078)	0.33 (0.055)
	Crisp	1.09	0.87	0.69	0.55	0.39
	Max	1.14 (0.046)	0.90 (0.037)	0.72 (0.029)	0.57 (0.023)	0.41 (0.017)
Spouse	Min	0.28 (0.053)	0.19 (0.036)	0.13 (0.024)	0.09 (0.016)	0.05 (0.009)
	Crisp	0.33	0.23	0.16	0.11	0.06
	Max	0.36 (0.021)	0.24 (0.014)	0.16 (0.010)	0.11 (0.007)	0.06 (0.004)
Children	Min	0.38 (0.072)	0.26 (0.049)	0.18 (0.033)	0.12 (0.022)	0.07 (0.012)
	Crisp	0.45	0.31	0.21	0.14	0.08
	Max	0.48 (0.028)	0.33 (0.019)	0.22 (0.013)	0.15 (0.009)	0.09 (0.005)
Sum of household	Min	1.60 (0.281)	1.19 (0.208)	0.90 (0.155)	0.68 (0.116)	0.45 (0.076)
	Crisp	1.88	1.40	1.05	0.80	0.53
	Max	1.97 (0.096)	1.47 (0.070)	1.11 (0.052)	0.84 (0.039)	0.55 (0.025)

[Unit: 10thousand yen/year]

Table 3. Result of benefit evaluation for the amenity river park

Distance until park		100m	300m	500m	700m	1000m
The head of household	Min	1.15 (0.180)	0.92 (0.143)	0.73 (0.114)	0.58 (0.090)	0.41 (0.064)
	Crisp	1.33	1.06	0.84	0.67	0.47
	Max	1.38 (0.046)	1.10 (0.037)	0.87 (0.029)	0.69 (0.023)	0.49 (0.017)
Spouse	Min	0.43 (0.070)	0.30 (0.047)	0.20 (0.032)	0.14 (0.021)	0.08 (0.012)
	Crisp	0.50	0.34	0.23	0.16	0.09
	Max	0.52 (0.021)	0.36 (0.014)	0.24 (0.010)	0.17 (0.007)	0.09 (0.004)
Children	Min	0.59 (0.095)	0.40 (0.065)	0.27 (0.044)	0.19 (0.029)	0.11 (0.016)
	Crisp	0.68	0.46	0.32	0.22	0.12
	Max	0.71 (0.028)	0.48 (0.019)	0.33 (0.013)	0.22 (0.009)	0.13 (0.005)
Sum of household	Min	2.17 (0.345)	1.61 (0.255)	1.20 (0.189)	0.90 (0.141)	0.59 (0.092)
	Crisp	2.52	1.87	1.39	1.04	0.69
	Max	2.61 (0.096)	1.94 (0.070)	1.44 (0.052)	1.08 (0.039)	0.71 (0.025)

[Unit: 10thousand yen/year]

$\ln[\alpha_1], \ln[\beta_1]$. However, as for setting of the fuzzy coefficient, we assume suppositionally to being $\tilde{A} = [0.9,\ 1.0,\ 1.1]$.

This set parameter $\tilde{A}$ implied that the fuzziness of visiting park demand of a person is $\tilde{u}_R = [1.237,\ 1.375,\ 1.512]$. Where is $\tilde{u}_R$ means the visiting number for one month. Though $\tilde{A}$ has to be determined by survey data in fact, that is remained important task.

C. Result of benefit evaluation

Table 2 and 3 show the measured benefits that are occurred by constructing the riverside park or the amenity river park.

These benefits are calculated for each agents, the head of household, suppose and children, and for the distance from their home to the park. And the minimization, crisp and maximization value of benefits are shown. Where is that the ( ) in table 2, 3 indicate the difference between the crisp value and minimization or maximization value..

The person who enjoys more benefit is the head of household. The reason is more height of his time value. His enjoying benefit is about 13.3 [thousand yen/year] for the constructing amenity river park. And for the $\tilde{A}$

Table 4. Social net benefit

	Riverside park	Amenity park
Min	5.37 (0.93)	7.21 (1.14)
Crisp	6.30	8.35
Max	6.61 (0.31)	8.66 (0.31)

[Unit: 100million yen/year]

which has the width of 10%, he loses benefit about 1.8 [thousand yen/year] on minimizing benefit case.

On the other hand, he is probable to obtain only benefits of 0.46 [thousand yen/year]. This reason for this measuring difference is considered that we adopted the necessity index for the benefit evaluation.

The social net benefits calculated as the number of household 55,627 are shown in table 4.

The SNB of the riverside park construction is 6.3 [100million yen/year] on the state of crisp. the width of benefit was from 5.37 to 6.61 [100million yen/year]. It is probable to lose the much benefit according to each case.

For constructing the amenity river park, the SNB is 8.35[100million yen/year]. Though much benefits are occurred, it is probable to lose 1.14 [100million yen/year] for the minimization benefit.

V. CONCLUSION

In the past research on the benefit evaluation, it is little considered on the fuzziness in the people behaviors. If the fuzziness is hidden in the people's activities, it is probable to lose confidence on the evaluated benefit.

In this paper, we built the benefit evaluating model in the case of the people activities including some fuzziness. In there, we introduced the indirect fuzzy utility function, and defined the benefit based on the concept of necessity measure for the fuzzy utility levels.

In the case study, the benefits for construction of the riverside park in Yamazaki River were measured. For the riverside park, social net benefit was 6.3 [100million yen/year] for crisp. And the width of benefit was from 5.37 to 6.61. There is the difference of about 1.2 [100million yen/year]. For the amenity river park, the SNB was 8.35 [100million yen/year] for crisp. The width was from 7.21 to 8.66, and the difference was 1.4[100million yen/year].

The remained task is expansion to the fuzzy general equilibrium model. Our model is partial equilibrium model for the recreation market. But, actually, economic activities are influenced by other many activities or economic circumstance. It is necessary to solve their problems by the general equilibrium.

Next task how to set the fuzzy utility function. Here, we only introduced $\tilde{A}$ into the utility function. It is necessary to argue including the reconsideration of utility function form.

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