THE ECONOMIC EVALUATION OF THE ENVIRONMENTAL CONTROL POLICIES
WITH AN OPTIMAL ECONOMIC GROWTH THEORY

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1. INTRODUCTION

In the recent argument to environmental problems, it is apprehended that the activities of future generation may be threatened by consuming excess natural resources or discharging more wastes and pollutants at present. It is necessary to examine the natural resources utilization or the control of wastes and pollutants with considering not only present generations’ activities but also future generations’ in order to achieve the sustainable development. In the field of the economics, the studies on an optimal economic growth theory have been advanced to analyze the continuous time dynamic growth and their welfare implication, and it has been applied to argue the dynamic environmental problems.

We pick up the problems of the waste discharge among some serious environmental problems. The waste problems are afraid to be more serious in Japan that is a very small country. In this paper, first of all, we build the general equilibrium model on the static framework, in which the waste discharge as well as agents’ activities are described. Next, we extend the static model to dynamic model on the framework of an optimal economic growth model. The waste disposal industry is introduced into the dynamic model, and we examine the control of waste problems.

2. PRELIMINARY STATIC GENERAL EQUILIBRIUM MODEL

2.1. Assumptions
The static model has the following assumptions.
1) Four agents exist in the economy: representative household, Composite good industry, waste disposal industry and absentee landowner (Fig.1).
2) We focus on the municipal wastes discharged by household. The amount of the municipal waste depends on the household composite consumption.
3) The waste disposal industry produces service to dispose the municipal wastes. The amount of the disposed waste is determined by the waste disposal industrial production, the rest of wastes other than disposed wastes are conveyed to the sanitary landfill.

![Fig.1 Mutual relation of the agents](image-url)

### 2.2. Household

Behavior formulation of the utility maximization:

\[
V = \max_{x_M, x_W, x_L, s} U(x_M, x_W, x_L, s, Q) \tag{1.a}
\]

s.t.

\[
p_M x_M + p_W x_W + p_L x_L = wL + rK + \pi_M + \pi_W + \pi_L \tag{1.b}
\]

\[
\Omega = L + s \tag{1.c}
\]

Where, \( U \): the utility function, \( x_M \): the composite consumption, \( x_W \): the consumption of disposal service of municipal wastes, \( x_L \): the land service consumption, \( s \): the leisure consumption, \( Q \): environmental quality associated with wastes, \( L \): the labor supply, \( K \): the initial capital stock endowment, \( \pi_M \): the divided income of composite good industrial profit, \( \pi_W \): the divided income of waste treatment industrial profit, \( \pi_L \): the divided income of absentee landowner’s profit, \( \Omega \): the total available time, \( p_M \): the price of composite good, \( p_W \): the price of disposal service of municipal wastes, \( p_L \): the land rent, \( w \): the wage rate, \( r \): the capital return rate, \( V \): the indirect utility.

Differential of the indirect utility:

\[
dV = \lambda [d \pi_M - x_M dp_M - x_W dp_W - x_L dp_L + Ldw + Kdr + d\pi_M + d\pi_W + d\pi_L] + \frac{\partial V}{\partial Q} dQ \tag{2}
\]

Where, \( \lambda \): the Lagrangean multiplier.

### 2.3. Composite good industry

Behavior formulated as profit maximization:

\[
\Pi_M = \max_{y_M, l_M, k_M, x_M} p_M y_M - (w l_M + r k_M + p_L x_M^M) \tag{3.a}
\]

s.t.

\[
y_M = f^M(l_M, k_M, x_M^M) \tag{3.b}
\]

Where, \( \Pi_M \): the profit, \( y_M \): the production, \( l_M \): the labor input, \( k_M \): the capital input, \( x_M^M \): the land
input, \( f^W(\cdot) \): production function.

Differential of profit:

\[
d\Pi_M = d\pi_M = y_M dp_M - l_M dw - k_M dr - x_M^W dp_L
\]  

(4)

2.4. Waste disposal industry

Behavior formulated as profit maximization:

\[
\Pi_w = \max_{y_w, l_w, k_w} p_w y_w - \left( w l_w + r k_w \right) 
\]  

(5.a)

s.t. \( y_w = f^w(l_w, k_w) \)  

(5.b)

Where, \( \Pi_w \): the profit, \( y_w \): the production, \( l_w \): the labor input, \( k_w \): the capital input, \( f^w(\cdot) \): production function.

Differential of profit:

\[
d\Pi_w = d\pi_w = y_w dp_w - l_w dw - k_w dr
\]  

(6)

2.5. Waste discharge

Here, we focus on the municipal wastes discharged by household. The amount of the municipal waste is assumed to be proportional to the household composite consumption.

The municipal wastes discharged by household are disposed by the waste disposal industry. The amount of the disposed waste assumed to be proportional to the waste disposal industrial production, the rest of wastes other than disposed ones are conveyed to the landfill.

The amount of sanitary landfill:

\[
Z = \alpha x_M - \beta y_w 
\]  

(7)

Where, \( Z \): the amount of sanitary landfills, \( \alpha \): marginal waste discharge, \( \beta \): marginal disposed waste.

2.6. Absentee landowner

Definition of the absentee landowner profit:

\[
\Pi_L = p_L \left[ x_L - x_L^w(Z) \right] 
\]  

(8)

Where, \( \Pi_L \): the profit, \( x_L \): the total available land, \( x_L^w \): the area of sanitary landfills.

Differential of profit:

\[
d\Pi_L = d\pi_L = x_L dp_L - x_L^w dp_L - p_L dx_L^w
\]  

(9)
2.7. Market clearing conditions

Composite good market: \( y_M = x_M \) \hspace{1cm} (10.a)

Waste disposal service market: \( y_W = x_W \) \hspace{1cm} (10.b)

Labor market: \( L = l_M + l_W \) \hspace{1cm} (10.c)

Capital market: \( K = k_M + k_W \) \hspace{1cm} (10.d)

Land market: \( x_L - \lambda_L^W(Z) = x_L + x_L^M \) \hspace{1cm} (10.e)

3. THE DEFINITION OF ECONOMIC LOSS CAUSED BY WASTE DISCHARGE

The definition of the economic loss caused by the waste discharge in the form of equivalent variation:

\[
V(\{p_A^M, p_A^W, p_L^A, I^A + \tilde{E}_L, Q^A\}) = V^B
\]

(11)

Where, \( \tilde{E}_L \): the economic loss caused by the waste discharge, \( A, B \): superscripts denoting without increase of waste discharge and with increase of one, respectively, \( I \): the full income \( (= w\Omega + rK + \pi_M + \pi_w + \pi_L) \).

The economic loss in differential form:

\[
EL = \lim_{\lambda \rightarrow B} \frac{\partial e}{\partial V} dV \hspace{1cm} (12.a)
\]

\[
= \lim_{\lambda \rightarrow B} \frac{\partial e}{\partial V} \left\{ x_M dp_M - x_W dp_W - x_L dp_L + Ldw + Kd\pi_M + d\pi_w + d\pi_L \right\} + \frac{\partial V}{\partial Q} dQ \hspace{1cm} (12.b)
\]

\[
= \lim_{\lambda \rightarrow B} \frac{\partial e}{\partial I} \left\{ x_L dp_L - Ld\pi_L + \frac{\partial I}{\partial Q} dQ \right\} \hspace{1cm} (12.c)
\]

The equation (12.c) implies that the economic loss of waste discharge is caused by the decrease of the land area supplied to the household and industry, and the deterioration of the environmental quality associated with wastes.

4. THE OPTIMAL ECONOMIC GROWTH MODEL

4.1. Assumptions

The basic assumptions of an optimal economic model are same with the ones of the static model.

1) The representative household is assumed to live infinitely.
2) The representative household maximizes the discounted sum of utility. But the industries maximize the profit at each point in time.
3) As for the land, the representative household has all of the total available land.
4.2. Household

Behavior formulation on the maximization of the discounted sum of utility:

\[ V = \max_{x'_{M}, x'_{W}, x'_{L}, s'} \int_{0}^{\infty} U(x'_{M}, x'_{W}, x'_{L}, s', Q'(y'_{M})) \exp(-\rho t) dt \]  
\[ \text{s.t.} \ K' = w' L' + r' K' + p'_{L} \left( x_{L}' - x_{L}'(Z) \right) - (p'_{M} x_{M}' + p'_{W} x_{W}' + p'_{L} x_{L}') - \delta K' \]  
\[ \Omega = L' + s' \]  

(13.a)  
(13.b)  
(13.c)

Where,  \( V \): the sum of discounted utility,  \( U \): the instantaneous utility,  \( \rho \): the subjective discount rate,  \( K (= dK/dt) \): the accumulation of the capital stock,  \( \delta \): the capital depreciation rate.

The behavior of industries and the market clearing conditions are same that the ones of the static model.

The current value Hamiltonian:

\[ H' = \left\{ U(x'_{M}, x'_{W}, x'_{L}, s', Q'(y'_{M})) \right\} \]
\[ + \lambda' \left[ w' \Omega + r' K' + p'_{L} \left( x_{L}' - x_{L}'(Z(x'_{M}, x'_{W})) \right) - p'_{M} x_{M}' - p'_{W} x_{W}' - p'_{L} x_{L}' - w' s' - \delta K' \right] \exp(-\rho t) \]  

(14)

The conditions to maximize the current value Hamiltonian:

\[ \frac{\partial H'}{\partial x'_{M}} = \frac{\partial U'}{\partial x'_{M}} - \lambda' \left[ p'_{M} + p'_{L} \frac{\partial x'_{L}}{\partial x'_{M}} \right] = 0 \]  
(15.a)

\[ \frac{\partial H'}{\partial x'_{w}} = \frac{\partial U'}{\partial x'_{w}} - \lambda' \left[ p'_{W} + p'_{L} \frac{\partial x'_{L}}{\partial x'_{w}} \right] = 0 \]  
(15.b)

\[ \frac{\partial H'}{\partial x'_{L}} = \frac{\partial U'}{\partial x'_{L}} - \lambda' p'_{L} = 0 \]  
(15.c)

\[ \frac{\partial H'}{\partial s'} - \lambda' w' = 0 \]  
(15.d)

\[ \dot{K}' = \frac{\partial H'}{\partial \lambda'} \]
\[ = w' \Omega + r' K' + p'_{L} \left( x_{L}' - x_{L}'(Z(x'_{M}, x'_{W})) \right) - p'_{M} x_{M}' - p'_{W} x_{W}' - p'_{L} x_{L}' - w' s' - \delta K' \]  
(15.e)

\[ \dot{\lambda}' - \rho \lambda' = \frac{\partial H'}{\partial K'} \]
\[ = -\lambda' [r - \delta] \]  
(15.f)

\[ \lim_{t \to \infty} \lambda' K' \exp(-\rho t) = 0 \]  
(15.g)

Equation (15.a-d): The first-order conditions of the instantaneous utility. It means that the marginal utilities of the consumption good/service equal the multiplying the marginal value of one additional unit of capital by its prices, respectively. As for the
composite good and the waste disposal service, its price is modified by the current value of the changing land area supplied to the household and industry.

Equation (15.e): The function of the capital accumulation.
Equation (15.f): The transversality condition.

5. CONCLUSION

We build an optimal economic growth model in which the waste discharge and its disposition are introduced.

1) The economic loss of waste discharge is caused by the decrease of the land area supplied to the household and industry, and the deterioration of the environmental quality associated with wastes.
2) We introduced the conditions to maximize the discounted sum of utility in that the economic loss of waste discharge was included.

We will measure the optimal path of consuming goods/services and controlling waste discharge through the simulation analysis.

REFERENCES


